A DYNAMIC STOCHASTIC MODEL OF A FISH POPULATION

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A B S T R A C T

By considering a milkfish population growing in a well-managed pond where no births occur and where in-migration and out-migration of fish are not allowed, a dynamic stochastic mortality model of a biological population is developed which may also apply to biological populations other than a fish population. The process occurring in the pond is described in two equivalent ways:

i) In terms of the basic differential equations

$$\dot{\underset{\sim}{q}}(t) = \underset{\sim}{A}(t)\ \underset{\sim}{q}(t)$$

where $\underset{\sim}{q}(t)$ is a vector of probabilities associated with the size of the population, $\dot{\underset{\sim}{q}}(t)$ is a vector of first derivatives with respect to time of these probabilities, and $\underset{\sim}{A}(t)$ is a matrix involving the parameters used to described the process.

ii) In terms of the Kolmogorov forward differential equations

$$\dot{\underset{\sim}{P}}(\tau, t) = \underset{\sim}{P}(\tau, t)\ \underset{\sim}{A}'(t)$$

where $\underset{\sim}{A}(t)$ is the same matrix as in i) and $\underset{\sim}{P}(\tau, t)$ is a matrix of transition probabilities.

When the parameters defining the process are taken at a fixed time, a recursive solution to the system of differential equations is derived and certain properties of the process are obtained. Explicit solutions to the system of differential equations are obtained under more simplifying assumptions on the parameters. Maximum likelihood estimators of the parameters are also derived under certain assumptions regarding the times of entrance of the diseased states and the healthy states.