ON THE ASYMPTOTIC PROPERTIES OF KERNEL-TYPE ESTIMATORS FOR FUNCTIONALS OF DENSITY FUNCTION

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Abstract

Strong consistency and asymptotic normality of kernel-based estimates, $H(f_n, f_n', f_n'', \ldots, f_n^{(r)})$ for a general class of functionals of density and its derivatives of the form $H(f, f', f'', \ldots, f^r) = \int h(f, f', f'', \ldots f^{(r)})dx$ are studied. Differentiability of this class of functionals is discussed and evaluated to verify both the almost sure convergence and asymptotic normality of the estimates. Schuster's lemma on the strong convergence of kernel estimates for density and its derivatives is extensively used to see if strong convergence of the kernel type of estimates for the functionals follows from the same property of the kernel estimates of density. The von Mises theory for asymptotic distribution is employed to show asymptotic normality of the estimates for functionals. The nonlinear functionals of density such as the efficacy parameter $\int f^2dx$, the Shannon entropy, $\int f\log f dx$ and the Fisher information $\int (f')^2/f\ dx$ provide as applications.