Covariance Theory of Rotation-Scale-Reflection-Invariant Random Fields

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Abstract

A strictly rotation-scale-reflection-invariant (RSR-invariant) random field is a spatial random function whose finite-dimensional distributions are invariant to rotations, changes in scale, and reflection of the plane. Covariance theory for this type of random field is sought, and thus, the investigation focuses mainly on second-order properties. In particular, the mean and covariance functions are examined in detail. This leads to the definition of a weakly RSR-invariant random field, whose mean function is constant, and whose covariance function depends only on the angle measure and minimal norm ratio between points on the plane. Weakly RSR-invariant random fields comprise the main subject of this research, and it is proposed that they are appropriate models for real-world phenomena that possess the above-mentioned invariance. The properties of a (weakly) RSR-invariant random field, including geometric structure, continuity, and spectral theory, are studied. The connection of RSR-invariant random fields to homogenous random fields and self-similar random fields are established. Finally, basic statistical analyses of RSR-invariant random fields are discussed, particularly the estimation of the mean and covariance functions, as well as the spatial prediction of the random field at unsampled locations.