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TiVEx-POT Models**

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ABSTRACT

Financial institutions hold risks in their investments that can potentially affect their ability to serve their clients. For banks to weigh their risks, Value-at-Risk (VaR) methodology is used, which involves studying the distribution of losses and formulating a statistic from this distribution. From the myriad of models, this paper proposes a method of formulating VaR using the Generalized Pareto distribution (GPD) with time-varying parameter through explanatory variables (TiVEx) - peaks over thresholds model (POT). The time varying parameters are linked to the linear predictor variables through link functions. To estimate parameters of the linear predictors, maximum likelihood estimation is used with the time-varying parameters being replaced from the likelihood function of the GPD. The test series used for the paper was the Philippine Peso-US Dollar exchange rate with horizon from January 2, 1997 to March 13, 2009. Explanatory variables used were GARCH volatilities, quarter dummies, number of holiday-weekends passed, and annual trend. Three selected permutations of modeling through TiVEx-POT by dropping other covariates were also conducted. Results show that econometric models and static POT models were better-performing in predicting losses from exchange rate risk, but simple TiVEx models have potential as part of the VaR modeling philosophy since it has consistent green status on the number exemptions and lower quadratic loss values.

Keywords: Value-at-Risk, Extreme Value Theory, Generalized Pareto Distribution, Time-Varying Parameters, Use of Explanatory Variables, GARCH modeling, Peaks-over-Thresholds Model

Estimating Value-at Risk (VaR) using TiVEx-POT Models

Peter Julian A. Cayton¹, Mary Therese A. Lising² and Dennis S. Mapa³

I. Introduction

Financial institutions expose their assets and equities to different forms of risks that can potentially affect how they service their clients in providing financial products. To understand the nature of risks, research aimed to explain its behavior using financial standards, corporate ratings, and statistical techniques flourished and the modeling and measurement of risks had been as diverse as the factors of the phenomenon. To guide financial institutions in efficient risk management, financial regulators introduced guidelines and standards that would aid in measuring and preparing for risk.

The Bangko Sentral ng Pilipinas (BSP) has stated that in handling risk, financial institutions should have sufficient capital at hand. Being short of capital relative to risks possessed by the bank from its portfolios makes the bank susceptible to failure and ultimately to bankruptcy. To have inefficiently greater capital with respect to risks would be an opportunity cost to the institution to which the funds could have been used to expand assets. The BSP has set the bar for appropriate level of capital on levels of risk incurred through the risk-based capital adequacy ratio. It is the ratio of capital in equity and total weighted risk incurred by the institution. The BSP requires financial firms to keep their capital adequacy to ten percent, more conservative from the Basel II provisions (2004) of eight percent. Three main parts of risks are to be assessed by these institutions; (1) credit risks, which are risks incurred from lending to other identities, (2) market risks, which are incurred from holding assets susceptible to changes in market price, such as stock exchanges, currencies, and commodities, and (3) operational risks, which are risks incurred from uncertainties in internal processes,

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such as legal risks and electric failure risks (BSP Memo Circ. No. 538). Of these, market risks is our interest because of its risk management methodology is dependent on the study of financial econometrics and time series analysis. The BSP consider to main approaches in accounting for these risks, one of them of them is the internal models approach (IMA), which deals with the use of statistical modeling in assessing risks, or the mark-to-model method of valuation.

The BSP has outlined a series of guidelines concerning market risk management of financial institutions with respect to their acquisition of financial assets and products. The guidelines are summarized by the four steps in managing financial risks: identification, measurement, control, and monitoring. In all these steps, the importance of modeling or accounting market risk could never be emphasized further. The BSP also states that effective risk measurement methodologies as one of the four basic elements in sound risk management practices (BSP Memo. Circ. No. 544). Such is the reason for the search of the appropriate statistical methodology in risk measurement in the field of private corporations and the academe.

With the highlights on risks earlier discussed, this paper aims to use a different approach to measuring risks by minima returns using the Peaks-Over-Thresholds (POT) model and the Generalized Pareto distribution from Extreme Value Theory (EVT) and improving the flexibility of the procedure by introducing a deterministic model of time-varying parameters in the distribution from information on variables explanatory to the returns. Section I discussed an introduction to risk management and guidelines set by the BSP. Section II aims to give a background literature on the Value-at-Risk (VaR) framework started by Jorion (2000), to show the evolution of the VaR models from RiskMetrics to the proposed model of time-varying parameters-POT, and to show the comparison tests and procedures used to measure the conservatism, efficiency, and accuracy of VaR . Section III aims to show the descriptive and exploratory analysis of the information set and explanatory variables to be used in performing the procedure. Section IV aims to compare the efficiency and accuracy of the new model with other prevailing statistical methods through backtesting results in terms of occurrence of exceedances and magnitudes of exceedances. Section V concludes the results of

the paper and summarizes recommendations and stepping stones for further research into modeling with EVT.

II. Review of Related Literature

Value-at-risk (VaR) measurement is a prevailing market risk methodology which uses statistical procedures in probability and distribution theory and cites the minimum potential loss of holding or possessing assets susceptible to market price change in a given period of time and probability, holding some assumption on what is the distribution of losses. Simply describing, VaR statements are built in this format: “The firm can lose more than V pesos T days later with the chance of α ” (Suaiso, 2009).

Sourcing from Tsay (2005), VaR is defined in statistical terms. Assume d to be the time interval when risk is to be anticipated and measured. $\Delta V(d)$ is the random variable of change in value of the asset held by a firm, and $F_d(\cdot)$ is the cumulative distribution function of $\Delta V(d)$. We define the long-position VaR at forecast horizon d with probability α as:

$$\alpha = F_d(\text{VaR}) = \Pr[\Delta V(d) \leq \text{VaR}] \quad (\text{i})$$

In a long position, the asset owner loses asset values when the change in market price of his asset become negative, thus the losses are negative in value.

RiskMetrics® Modeling

The RiskMetrics methodology was developed by J.P. Morgan to perform VaR calculations. In its simple form, it assumes that the continuously compounded daily returns of a portfolio follow a conditional normal distribution. Riskmetrics assumes $r_t | F_{t-1} \sim N(\mu_t, h_t)$ where $r_t = \log$ returns; F_{t-1} = information set available at time $t-1$; $\mu_t = 0$ is the conditional mean of r_t and $h_t = \alpha h_{t-1} + (1-\alpha)r_{t-1}^2$, $0 < \alpha < 1$ and h_t is the conditional variance of r_t . RiskMetrics defines 1-day VaR with $100(1-p)\%$ confidence level as,

$$\text{VaR} = \text{Amount of position} \times F^{-1}(p)h_{t+1}^{1/2} \quad (\text{ii})$$

The $F^{-1}(p)$ is the 100p% quantile of a standard normal distribution and h_{t+1} is the 1-step ahead forecast of the conditional variance given by $h_{t+1} = \alpha h_t + (1 - \alpha)r_t^2$. The limitations of the method are that return series are not normally distributed, with majority of series being possibly fat-tails, leading to an underestimation of an appropriate VaR for such assets (Tsay, 2005).

Econometric Methods (ARMA-GARCH models)

The measurement of VaR in terms of the mean ARMA (autoregressive moving average) specification and a GARCH (generalized autoregressive conditional heteroscedasticity) model can be facilitated and distributional assumption on the standardized errors can be adjusted or made robust. ARMA modeling facilitates the factoring in of serial autocorrelation, GARCH modeling is flexible as it accounts volatility clustering and non-constant variances natural to return series (Engle, 2004) and the method is easy since substitution of values is only necessary to measure VaR. The risk equation of holding 1-unit of an asset using econometric method reduces to the equation below.

$$VaR = \hat{r}_{t+1} - F^{-1}(p)h_{t+1}^{1/2} \quad (iii)$$

The \hat{r}_{t+1} = the estimated mean at time $t+1$ modeled by the ARMA(p,q) specification, $h_{t+1}^{1/2}$ = conditional variance model, either a deterministic GARCH model or a stochastic volatility model (Taylor, 1986), and $F^{-1}(p)$ is the quantile function at probability p . The quantile function depends on the assumed error distribution. Common are standard normal, standard t, and the generalized error distribution (Tsay, 2005). The risk-return premium models such as GARCH-in-Mean specification can be used as VaR models, as considered by Suaiso (2009).

Extreme Value Theory in VaR Measurement

Extreme value theory (EVT) bases itself on the study of extreme occurrences and the probability structure. The theory focuses itself to the distribution of the tails of a random sample as the sample size increases. It models extreme risks by observing the behavior of the return distribution at the tails (Tsay, 2005). The theory involves two branches of measuring

extreme returns: (1) Block Maxima, or unconditional method, and (2) Peaks-over-Threshold method (POT), also called the conditional method.

Block Maxima

Tsay (2005) discussed the overview of the rationale of the block maxima technique. Let us suppose that there is an information set of independent returns $\{r_t\}$ of T days taken from a distribution $F(x)$. Denoting $r_{(1)}$ as the minimum order statistic, which may be taken as a VaR for the long position, its distribution function $F_1(x)$ would reduce to:

$$F_1(x) = 1 - [1 - F(x)]^T \quad (\text{iv})$$

The distribution reduces to a degenerate form when the sample size goes very large and such information is irrelevant. To understand the behavior of tail returns, we should transform the return series to a set of standardized minimum $r_{(1)^*} = (r_{(1)} - \beta_n) / \alpha_n$ with β_n is a location parameter and $\alpha_n > 0$ is a scaling factor that would avoid the convergence of the limiting distribution to degeneracy. The limiting distribution of this transformed minimum as the sample size gets larger is said to be the *generalized extreme value distribution* (GEV) with tail parameter ξ_n . Its distribution is defined by the equation below:

$$F_{r^*}(x) = \begin{cases} 1 - \exp[-(1 - \xi_n x)^{-1/\xi_n}] & \text{if } \xi_n \neq 0, \\ 1 - \exp[-\exp(x)] & \text{if } \xi_n = 0. \end{cases} \quad (\text{v})$$

To estimate the parameters α_n , β_n , and ξ_n , numerous sample minimums should be observed from the information set, dividing the series in g equal sub-samples of size n . From these subsets, the minimum is taken and the parameters are estimated by either maximum likelihood method using GEV or the regression method of sub-sample minimums. The parameters are marked with subscript n since their values depend on the sub-sample size.

To measure the 100(1-p)% VaR after using the Block Maxima, assuming a short position, the following equation is used:

$$VaR = \begin{cases} \beta_n + \frac{\alpha_n}{\xi_n} \{1 - [-n \ln(1-p)]^{-\xi_n}\} & \text{if } \xi_n \neq 0 \\ \beta_n + \alpha_n \ln[-n \ln(1-p)] & \text{if } \xi_n = 0 \end{cases} \quad (\text{vi})$$

Extensive research and confirmations on the asymptotic distribution of extreme returns from companies listed in the New York Stock Exchange converging to GEV were undertaken by Longin (1996) using graphical methods backed-up by the limiting distribution's theoretical properties. The disadvantage of the block maxima is its high sample requirement: (1) large n to supply a sufficient minimum for each sub-sample, and (2) enough g sub-sample minimums to make the Generalized Extreme Value distribution approximation sufficiently accurate. These restrictions make the block maxima method a difficult model to comply.

Peaks-over-Thresholds

Instead of modeling through minimum or maximum values, a paradigm using exceedance events from a feasible threshold $\eta > 0$ and the magnitude of exceedance was proposed by Smith (1989). Since differences with respect to threshold were being used in modeling, the method was called *peaks over thresholds approach* (POT). In modeling using this approach, a pair of observations are recorded: (1) the time of exceedance t_i when $r_{t_i} \geq \eta$ for short financial position holding, and (2) the magnitude of exceedance $r_{t_i} - \eta$. Common in literature, POT models use positive thresholds that estimate the maximum, such as texts by Bystrom (2002) and the programming tutorial of Gilleland and Katz (2005). The approach is called the conditional model since the conditional distribution of $r_{t_i} - \eta$ given that $r_{t_i} \geq \eta$ is being used. Tsay (2005) provides the derivation of this distribution and shows that it reduces to the *generalized Pareto distribution* (GPD) defined by the following equation:

$$F_{\xi, \sigma_\eta}(x) = \begin{cases} 1 - \left[1 + \frac{\xi x}{\sigma_\eta} \right]^{-1/\xi} & \text{for } \xi \neq 0, \\ 1 - \exp \left[-\frac{x}{\sigma_\eta} \right] & \text{for } \xi = 0, \end{cases} \quad (\text{vii})$$

The parameter ξ is defined as the shape parameter of the distribution and σ_η is defined as the scale parameter.

The conundrum that a researcher tackles in using POT models is what threshold to use; Gilleland and Katz (2005) provide a graphical method to selecting thresholds, such as the *mean excess function*, which is also featured in Tsay (2005), and threshold fitting of

parameter estimates, where the stability of parameter estimates given an interval of potential thresholds is used as criterion. Other methods still exist in literature, and the researcher is not bounded by such in self-assigning a threshold by convenient rational judgment or by just simple research constraint.

To measure $100(1-\alpha) \% VaR$ for the long position, the equation is a simple substitution of the following quantities derived using estimation results of maximum likelihood:

$$VaR_{(1-\alpha)}(1) = \eta - \frac{\sigma_\eta}{\xi} \left[1 - \left\{ \frac{T}{N_\eta}(\alpha) \right\}^{-\xi} \right] \quad (\text{viii})$$

The N_η is the number of exceedances from the threshold in the sample period and T is the sample size of the evaluation.

As attractive as the EVT methods are in modeling tail distribution, there are some problems in using them; they do not account seasonal or time dependence in the means and variances of time series data which is central and important for potent statistical forecasting. Two methods sprung out as a solution to the problem: (1) model time dependence first of the series through time series analytics then modeling the standardized or adjusted residuals on an extreme value distribution, and (2) the use of explanatory variables to create a model of time-varying parameters (McNeil and Frey, 2000; Smith 2003).

ARMA-GARCH-POT (AGPOT) Modeling of VaR

McNeil and Frey (2000) introduced a different procedure that made the POT model more “time-adaptable.” It combines the idea of econometric methods and extreme values modeling by a simple two-step modeling procedure. They suggested a VaR solution of this manner, which in statistical terms is defined by:

$$VaR(1) = \mu_{t+1} + h_{t+1}^{1/2} VaR(Z_{t+1}) \quad (\text{ix})$$

The μ_t is the conditional mean of the return series specified and forecasted by an appropriate ARMA model and the h_t is derived from the appropriate GARCH model for the conditional variance of the returns. $Z_t = (r_t - \mu_t)/h_t^{1/2}$ is the standardized residual of the series, to which the kurtosis is tested if thick tails are still observed. If excess kurtosis exhibits normality or if the

distribution of Z_t fits an error distribution, then a quantile from that error distribution is sufficient to cover VaR and substitute the $VaR(Z_{t+1})$ term. Otherwise, a POT model is fitted to the standardized residuals which would estimate $VaR(Z_t)$ using (vii) and thus formulate a $VaR(1)$. Suaiso (2009) has concluded research in the AGPOT for interest rate risk from secondary bond markets to be a conservative, accurate, and efficient model.

Use of Explanatory Variables (TiVEx-POT)

Besides the AGPOT model for measuring VaR, Tsay(2005), Coles (2001) and Smith (2003) proposed a model of the POT that uses a different concept of flexibility through time. They account that the shortcomings of the first EVT models are that they only utilizes information from the original series and does not account information from explanatory variables. These variables may be able to describe the behavior of the log returns and the new extreme value theory approach to VaR computations can easily take into consideration these additional features of flexibility and information. For asset returns, a volatility index is an example of an explanatory variable. The explanatory variables can be used by postulating that the two parameters ξ , and σ in the GPD are time-varying and are linear functions of the explanatory variables and new parameter coefficients of these variables (Tsay, 2005; Coles, 2001). Tsay (2005) suggests that these explanatory variables should be known prior to time of evaluation t , such as time dummies (e.g., trends, quarters, months, weekdays, number of holidays), and measures of past volatility indices (e.g., GARCH variances, number of exceeding observations in the past 2 weeks) and other variables wished to be explored (e.g., panic buying effects, crisis adjustments). This time-varying-parameters-through-explanatory-variables POT (TiVEx-POT) is the proposed model of this paper using time variables and volatility measures as suggested by Tsay (2005) and Smith (2003).

In statistical terms we create a new model of GPD parameters $\sigma_{t,\eta}$ and ξ_t linearly dependent to explanatory variables $\mathbf{x}=(x_{1t},x_{2t},\dots,x_{pt})$ and $\mathbf{y}=(y_{1t},y_{2t},\dots,y_{qt})$ and linked through functions g_1 and g_2 respectively:

$$\begin{aligned} g_1(\sigma_{t,\eta}) &= \sigma_0 + \sigma_1 x_{1t} + \dots + \sigma_p x_{pt} = \sigma_0 + \boldsymbol{\sigma}' \mathbf{x}_t \\ g_2(\xi_t) &= \xi_0 + \xi_1 y_{1t} + \dots + \xi_q y_{qt} = \xi_0 + \boldsymbol{\xi}' \mathbf{y}_t \end{aligned} \quad (\text{x})$$

The explanatory variables used between two parameters may be of the same set or of different sets; one parameter may be fixed as independent through time while the other varies. The link between the linear predictors and the time-varying parameter may be any one-to-one elementary function, such as the identity link (i.e., $g_1(u)=u$) or the logarithm link (i.e., $g_1(u)=\log_e u$). The scale link function is commonly the logarithm link since it restricts the scale parameter to positive values while the identity link is sufficient for the shape parameter. Ultimately, there are no limitations in the possibilities to be used in TiVEx-POT. There is no error term in the model of the time-varying parameters, so the model is said to be “deterministic.” Also, the linear coefficients are the new parameters, not σ_t and ξ_t , to be estimated in the GPD likelihood function using iterative nonlinear maximum likelihood estimation (MLE). Observations used for the MLE procedure are those that exceed the assigned threshold and explanatory information from these observations.

After estimating the linear parameters of the time-varying GPD model, the measurement of the short position VaR is derived with the new altered equation below:

$$VaR_{(1-\alpha)}(1) = \eta + \frac{\sigma_{t,\eta}}{\xi_t} \left[\left\{ \frac{T}{N_\eta}(\alpha) \right\}^{-\xi_t} - 1 \right] \quad (\text{xi})$$

To measure long position VaR would require to negatively transforming the data for the estimation and measurement using the equation above.

A procedure is suggested below in implementing TiVEx-POT modeling for VaR:

1. Gather the information set and the explanatory variables to be used to vary the parameters of the GPD. A criterion for selecting variables is that they should be known prior to time $t+1$, e.g., time indicators, GARCH volatility indices, past returns and volatilities of other series shown in theory or to be explored to have an effect to

the return series of interest, and other variables. Since research on non-stationary or co-integrating explanatory variables on GPD parameters are not yet pursued as of the time of the paper, use of stationary series is a conservative choice.

2. Set a threshold suitable for modeling. As said earlier, different threshold selection techniques flood the literature of EVT, but anyone can impose a threshold of their choosing, such as this paper selected the 0.01 logarithm difference of prices as threshold for every estimation period due to constraints in the number of observations necessary for parameter estimation. Impose constraints on the time-varying parameters using link functions, such as logarithm and identity functions.
3. Take the observations and all connected information on them which exceeds the set threshold. These values will be used as the sample for the estimation procedure. Perform initial estimation using MLE procedures to derive the estimates and the standard errors.
4. Check model assumptions on the GPD distribution using quantile-quantile and probability-probability plots of observed and fitted values for fit. Check for significance testing of explanatory variables and refine the model by re-specification procedures such as variable selection or reduction, transformation of independent variables, and changing the link functions.
5. When a final model has been adopted, gather the parameter estimates necessary for measurement and substitute them to the VaR equation. Habitually maintain and improve statistical model for better time flexibility and updating.

Backtesting VaR models through Exceedances: Basel Standards

The Basel Committee on Banking Supervision (1996) has outlined a framework for banks that use the Internal Models Approach in measuring VaR. This framework deals with the number of failures a bank can possess using their models and penalties when these criteria are broken. A bank is required to report his model and results of evaluation for VaR exemptions, which are number of occurrences that the VaR was not sufficient in estimating loss (i.e, the number of days when loss is greater than the assigned VaR). A risk model, based from the Committee, should be able to sufficiently forecast losses in 99% of the trading days in a year. The number

of allowable VaR exemptions within a year is equal to one percent of the number of trading days, around 2 or 3 out of 250 per year. For a certain interval of VaR exemptions, a bank is classified into three zones: green zone, yellow zone, and red zone. The green zone means that the model of the bank is able to estimate the market risk of holding assets adequately within 99% of trading days. The yellow zone means that the bank's model may be slightly inadequate as the model is not fully able to adjust in some instances such as sudden spikes and market shocks. The 99% standard is possible to be achieved, but at a very low confidence. The red zone means that the bank's model for market risk is not able to satisfy the Committee's standard of 99%. The number of VaR exemptions and the bank's assigned zone within a year is the basis in which how much market risk capital is necessary in holding the asset for that day. The table below shows the number of VaR exemptions, the three zones, appropriate multiplier for market risk, and the cumulative probability of occurrence of exemptions.

Zone	Number of Exemptions	Scaling Factor for Market Risk Capital	Cumulative Probability
Green Zone	0	3.00	8.11%
	1	3.00	28.58%
	2	3.00	54.32%
	3	3.00	75.81%
	4	3.00	89.22%
Yellow Zone	5	3.40	95.88%
	6	3.50	98.63%
	7	3.65	99.60%
	8	3.75	99.89%
	9	3.85	99.97%
Red Zone	10 or more	4.00	99.99%

Source: Basel (1996)

Table 1. Classification in Zones and Appropriate Scaling Factors for Capital Requirement

Assessing VaR models through Exceedances: Likelihood Ratio Tests (LRTs)

Christoffersen (1998) developed a procedural system of LRTs that examines the ability of VaR models to be within the allotted standard of confidence. The procedure tests the following in sequential order: (1) if it is within allowable proportion of VaR exemptions (unconditional coverage), (2) if the VaR exemptions are separate and independent from one another, i.e., no clustering of exemptions (independence), and (3) if there is clustering with the proportion of VaR exemptions within allowable limits (conditional coverage). These LRTs are indicators of a model's degree of accuracy in predicting occurrences of loss.

Unconditional Coverage

The test for unconditional coverage tests if the number of VaR exemptions equal to the assigned proportion of allowable VaR exemptions. To test the null hypothesis of $\pi = p$, where π = proportion of VaR exemptions in a year, the test statistic is given below:

$$LR_{uc} = 2 \times \log \left[\frac{(1 - \hat{\pi})^{T-T_1} \hat{\pi}^{T_1}}{(1-p)^{T-T_1} p^{T_1}} \right] \square \chi_{(1)}^2 \quad (\text{xii})$$

The ML estimate of the proportion is $\hat{\pi}_1 = T_1 / T$, where T_1 = number of VaR exemptions within a year, and T = number of trading days per year. Rejection of the hypothesis should lead the researcher to refine or change his model for measuring market risk. Non-rejection means that the model is able to keep in the allowable proportion of VaR exemptions. The weakness of doing this test alone is that it does not account the possibility of clustering of exemptions, which can be caused by volatile return series. The next test for independence should be used to augment analysis on unconditional convergence.

Independence

The test for independence hypothesizes that the probability of an isolated VaR exemption is equal to the probability of 2 consecutive VaR exemptions. In statistical terms, the null hypothesis of the test is $\pi_0 = \pi_1$, where π_0 is the proportion of VaR exemptions preceded by non-exemption and π_1 is the proportion of VaR exemptions preceded by another VaR exemption. The test statistic for the hypothesis is shown below:

$$LR_{ind} = 2 \times \log \left[\frac{(1 - \hat{\pi}_0)^{T_{00}} \hat{\pi}_0^{T_{01}} (1 - \hat{\pi}_1)^{T_{10}} \hat{\pi}_1^{T_{11}}}{(1 - \hat{\pi})^{T_{00}+T_{10}} \hat{\pi}^{T_{01}+T_{11}}} \right] \square \chi_{(1)}^2 \quad (\text{xiii})$$

The needed values for the LR statistic are listed below:

T_{00} = number of two consecutive days without VaR exemption;

T_{10} = number of days without VaR exemption preceded by a day with VaR exemption;

T_{11} = number of two consecutive days with VaR exemptions

T_{01} = number of days with VaR exemption preceded by a day without a VaR exemption;

$\pi_0 = \frac{T_{01}}{T_{01} + T_{00}}$ = proportion of VaR exemptions preceded by non-VaR exemption;

$\pi_1 = \frac{T_{11}}{T_{11} + T_{10}}$ = proportion of two consecutive VaR exemptions.

$\pi = \frac{T_{01} + T_{11}}{T_{00} + T_{01} + T_{10} + T_{11}}$ = pooled estimate for proportion of VaR exemptions

If the null hypothesis is rejected, then the model is susceptible to clustering VaR exemptions (i.e., an occurrence of a VaR exemption would tend to be followed by another VaR exemption) and the model would be inefficient to high volatility clusters. Non-rejection of the hypothesis would mean that the VaR model is not susceptible to VaR clustering, however the probabilities π_0 and π_1 have to be tested if they are equal to the allowable proportion of VaR exemptions p . The next test for conditional coverage covers this hypothesis and finalizes our result in exceedance analysis.

Conditional Coverage

The conditional coverage test leads us to conclude if the two conditional VaR exemption probabilities in independence test (i.e., the π_0 and π_1) are equal to the failure rate p (i.e., the proportion of allowable VaR exemptions). It is a joint test of the two likelihood ratio tests discussed earlier. The null statistical hypothesis of the test is that if $\pi_0 = \pi_1 = p$. To test this hypothesis, the test statistic that shall be used is written below:

$$LR_{cc} = LR_{uc} + LR_{ind} \quad \square \quad \chi^2_{(2)} \quad \text{(xiv)}$$

Rejection of this hypothesis leads to conclude that the model is not adequate in maintaining allowable VaR exemptions and is susceptible to VaR exceedance clustering. Non-rejection of this hypothesis leads to be confident in the reliability of the VaR model in the aspect of predicting events of losses.

Evaluating VaR models through Magnitudes

The occurrence of loss is important to be predicted definitely, but of more interest is the magnitude of losses avoided. A VaR model should also be able predict huge losses that would be incurred in holding an asset. Engel, J. and M. Gizycki (1999) described that an appropriate VaR model should have three characteristics: (1) conservatism, (2) accuracy, and (3) efficiency. A conservative model is a model that produces high measures of risks relative to other models. A accurate model is a model that can par its measure with volatile price movements. An efficient model is a model that can provide a conservative estimate of risk, but not so high that it requires the bank a very high capital. The next series of measures were selected from the paper of Engel and Gizycki (1999) and Basel (1996) as appropriate measure of the three characteristics.

Conservatism: Mean Relative Bias (MRB)

This is an average measure of relative size, and thus of average conservatism, of VaR models from their mean model estimate. The larger the MRB, the more conservative the model is at measuring risk. Suppose that N VaR models are being compared in a forecast period starting from 1 to T . The MRB statistic for the i th model is evaluated using the formula below:

$$MRB_i = \frac{1}{T} \sum_{t=1}^T \frac{VaR_{it} - \overline{VaR}_t}{\overline{VaR}_t} \text{ where } \overline{VaR}_t = \frac{1}{N} \sum_{i=1}^N VaR_{it} \quad (xv)$$

Accuracy: Average Quadratic Loss Function (AQLF)

This measure takes into account the occurrence and magnitude of loss and gives greater penalty weights for models that are not able to predict large losses. The quadratic loss function for the i th model at time t that is used in evaluating AQLF for long position VaR for each model is shown below:

$$L(VaR_{it}, \Delta \log P_t) = \begin{cases} 1 + (\Delta \log P_t - VaR_{it})^2 & \text{if } \Delta \log P_t \leq VaR_{it} \\ 0 & \text{otherwise} \end{cases} \quad (xvi)$$

Efficiency: Market Risk Capital (MRC)

This measure assesses VaR model efficiency in the case of assigning the appropriate capital given the expected risk of holding an asset susceptible to market forces. The formula below derives the MRC:

$$MRC = \max \left(\frac{k}{60} \sum_{i=1}^{60} VaR_{t-i}, VaR_{t-1} \right) \quad (\text{xvii})$$

The k in the maxima function is the scaling factor set by the Basel Committee depending on the number of VaR exemptions a bank has incurred using internal models approach (See Table 1). This measure is analyzed with the conservative and accuracy statistics to finally assess its efficiency.

III. Data Source and Methodology

The test data used in this paper was the return series of the daily weighted average of the Philippine Peso-US Dollar (P-\$) Exchange Rate (Figure 1). The weights for the formulation of averages were based on the volume of currencies traded in the Philippine Dealings and Exchanges Corporation. This data is released from the official website of the Bangko Sentral ng Pilipinas, spanning January 1997-March 13, 2009 as gathered and managed by the Institute for Development and Econometric Analysis, Inc., and by the researchers. Studying exchange rates in financial econometrics is relevant especially in transactions for foreign financial instruments, such as dollar bonds, dollar debts, and stock investments in US markets (BSP, 2008; BSP Memo Circ. 538 Attachment).

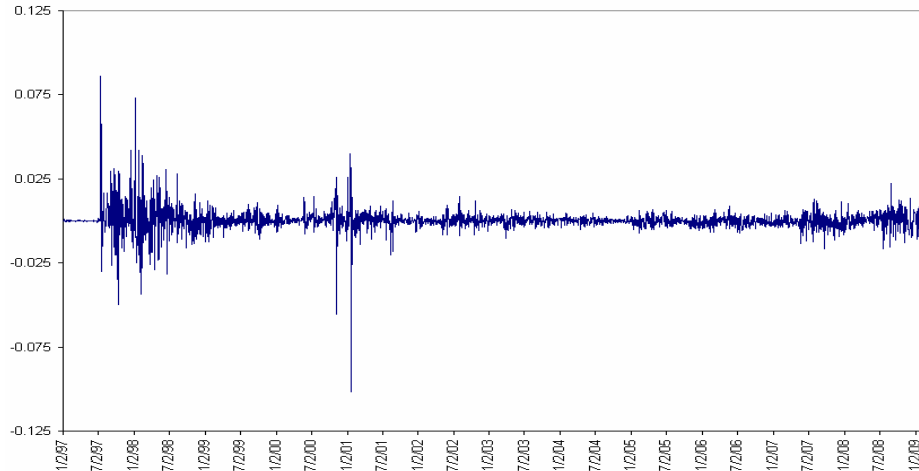


Figure 1. The Time Plot of Log-Returns from the P-\$ Exchange Rate
from January 1997 to March 2009

VaR Model Comparisons

In the paper, eleven models were considered for comparison between VaR methodologies: six econometrics models [GARCH (1, 1), EGARCH (1, 1, 1), TARCH (1, 1, 1), GARCH(1, 1)-in-M(Standard Deviation (Std. Dev.)), EGARCH(1, 1, 1)-in-M(Std. Dev), TARCH(1, 1, 1)-in-M(Std. Dev)] and five EVT models [Static POT, AR(2)-EGARCH(1, 1, 1)-POT, and TiVEx-POT, TiVEx-POT 2, TiVEx-POT 3, details of TiVEx models are in Table 2] for the following forecast years: (1) 2001 [when local political and US crises occurred], (2) 2005 [non-crisis year], (3) 2008 [most current full year; inciting financial crisis], and (4) 2009 [a continuing comparison of the models, studying the scenario of the global financial crisis]. Results of model parameters are preserved in the Appendix of the paper. Comparisons using the VaR assessment methods were calculated for each model for each year. The threshold for the static and TiVEx models is 1% percent change in exchange rate daily, while the AGPOT model has a threshold of 2 standard units for the residuals. In the TiVEx model, the logarithm link was used for the scale parameter and the identity link was used for shape to relate with linear predictors for each parameter. All explanatory variables and some selected permutations, which will be discussed later, were used in both scale and shape models. No insignificant linear predictors or variables were removed after performing the first run for each year and model, so as to maintain comparability through time comparisons.

TiVEx Model	Explanatory Variables	
	Scale	Shape
TiVEx-POT	Intercept, GARCH(1,1)-QMLE, Annual Trend, Quarterly Dummies, Holiday-Weekend	Intercept, Annual Trend, Quarterly Dummies, Holiday-Weekend
TiVEx-POT 2	Intercept, GARCH(1,1)-QMLE, Annual Trend, Quarterly Dummies	Intercept, GARCH(1,1)-QMLE, Annual Trend, Quarterly Dummies
TiVEx-POT 3	Intercept, GARCH(1,1)-QMLE	Intercept

Table 2. List of Explanatory Variables for TiVEx-POT Models

Descriptive Analysis of Test Series

From Figure 1, the return series is observed to have volatility clustering, in which large changes are followed by large changes and small changes are followed by small changes, such as in the years 1997-1999 and 2000-2001, when major crises occurred in the financial economy. Considering the measure of volatility is needed in modeling the exchange rate risk.

The distribution of the return series is highly leptokurtic, with kurtosis statistic equal 54 (see Figure 2). Fitting a normal error distribution is incorrect for this series; the t distribution was considered as a distribution for econometric VaR models. The sample mean is very small but still significantly different from zero, with Wald Chi-square statistic $[\bar{X}/SE(\bar{X})]^2$ equal to 3.16, with p-value of 0.0753. The distribution of returns is closely symmetric, thus a symmetric error distribution may be assumed, such as in this paper is the t distribution.

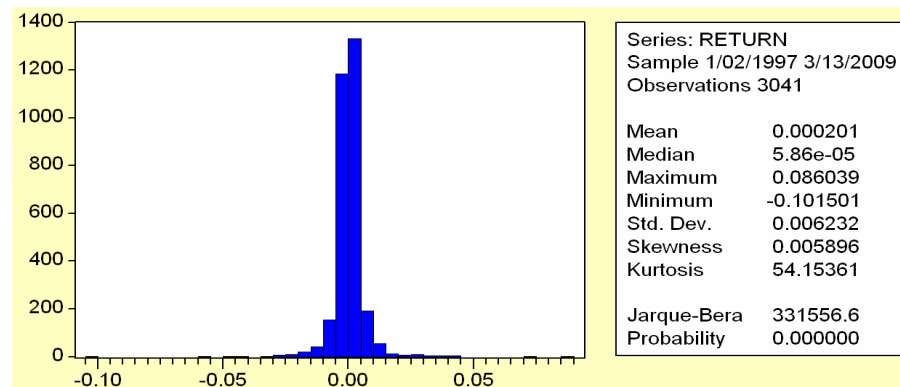


Figure 2. Histogram and Statistics of the Test Return Series

Explanatory Variables

The explanatory variables used for the TiVEx-POT model were the following: (1) annual deterministic trend, which is equal to $year_t-1992$ for the dataset, (2) GARCH (1, 1)-Quasi-MLE conditional variances, and (3) quarterly dummies.

Annual trend takes into account any trend in the changes of scale and shape in the distribution of returns with respect to the start of the data series. A positive or negative trend leads to conclude a slowly increasing or decreasing volatility respectively every year. A positive or negative trend in shape leads to conclude a slowly thickening or thinning of the tails respectively every year.

GARCH volatilities are used as implied quantitative indices which may be used as proxy variable of volatility in the series, as suggested by Tsay (2005) in his example of TiVEx-POT. Quasi-MLE was used as it assumes no distribution, but uses an altered version of the normal distribution to derive estimates. The Table 3 below shows the coefficients for the equations of the GARCH (1, 1)-QMLE for each forecast year.

In date variables, quarterly dummies were used. Quarterly dummy variables were used to account any seasonality in the data series. The first quarter was held as a baseline time. Holiday-weekend variables account for the effect of the number of holidays and weekends that pass before the trading day on the scale and shape of the distribution. Recent laws have changed in the policy of assigning holidays (RA 9492), and thus the effect of holidays on exchange rate volatility is being explored, such as the paper of Ariel (1990) on stock returns.

	2001	2005	2008	2009
C	1.20E-05	5.30E-06	3.70E-06	3.47E-06
resid(-1)	0.467367*	0.391562*	0.289287*	0.272183*
GARCH(-1)	0.459857*	0.52233*	0.601821*	0.623452*

Table 3. GARCH(1, 1)-QMLE Model Coefficients for Volatility Index

IV. Results and Discussion

The VaR models' conservatism, accuracy, and efficiency were assessed using the measures of backtesting and numerical indices expressed above. Results from these analyses and subsequent tables are drawn and explained below.

Model	Number of Exemptions				Average Quadratic Loss Function			
	2001	2005	2008	2009	2001	2005	2008	2009
GARCH	2	0	0	0	0.008	0	0	0
GARCH-in-M	2	0	0	0	0.008	0	0	0
EGARCH	2	0	0	0	0.00802	0	0	0
EGARCH-in-M	0	0	0	0	0	0	0	0
TARCH	2	0	0	0	0.008	0	0	0
TARCH-in-M	2	0	0	0	0.008	0	0	0
Static POT	3	0	0	0	0.012024	0	0	0
A-G-POT	18	0	3	0	0.072039	0	0.0121953	0
TiVEx-POT	8	0	3	0	0.032032	0	0.0121952	0
TiVEx-POT 2	6	0	2	0	0.024027	0	0.008130	0
TiVEx-POT 3	2	0	2	0	0.008016	0	0.008130	0

Table 4. VaR Exemptions and Average Quadratic Loss of Models for each Year

VaR Exemptions

Based from the number of VaR exemptions on Table 4, the econometric models performed across all years, even registering a green status in crisis period 2001. EGARCH-in-M was the top-performing POT model in the group with no exemptions in all years. Static POT is seen to be at par with econometric models registering only 3 exemptions for crisis period 2001 and none in other years. For time-dependent POT models, the simpler TiVEx-POT 3 outperforms the time-dependent models with lesser exemptions in the crisis period 2001, with 2 exemptions in 2001 and in 2008. TiVEx-POT 3 was the only time-dependent EVT model to sustain a green status in all years.

Likelihood Ratio Test Analysis

Similar to the analysis of the VaR exemptions in the section above, the likelihood ratio tests leads us to same conclusions. The GARCH models were performing better than the POTs, all of them either not rejecting the null hypothesis or testing for proportion is not necessary. In

the POT models, though Static POT was the best-performing even under inciting crisis, it is seen to be susceptible to volatility clustering that is common in crisis situations. TiVEx-POT 3 model was better-performing than most of the time-dependent models, and is better than Static POT for 2001.

	GARCH			GARCH-in-M			EGARCH			EGARCH-in-M		
	UC	IND	CC	UC	IND	CC	UC	IND	CC	UC	IND	CC
2001 A	INC	-		A	INC	-	A	INC	-	-	-	-
2005 -	-	-		-	-	-	-	-	-	-	-	-
2008 -	-	-		-	-	-	-	-	-	-	-	-
2009 -	-	-		-	-	-	-	-	-	-	-	-
	TARCH			TARCH-in-M			Static POT			A-G-POT		
	UC	IND	CC	UC	IND	CC	UC	IND	CC	UC	IND	CC
2001 A	INC	-		A	INC	-	A	R	-	R	-	-
2005 -	-	-		-	-	-	-	-	-	-	-	-
2008 -	-	-		-	-	-	-	-	-	A	INC	-
2009 -	-	-		-	-	-	-	-	-	-	-	-
	TiVEx-POT			TiVEx-POT 2			TiVEx-POT 3					
	UC	IND	CC	UC	IND	CC	UC	IND	CC	UC	IND	CC
2001 R	-	-		A	R	-	A	INC	-	-	-	-
2005 -	-	-		-	-	-	-	-	-	-	-	-
2008 A	INC	-		A	INC	-	A	INC	-	-	-	-
2009 -	-	-		-	-	-	-	-	-	-	-	-

R=Reject; A=Accept; INC =inconclusive, model shows proportions but cannot be tested;

“-“=cannot be tested.

Table 5. LRT Results for VaR Models for each Year

Conservatism: MRB Analysis

Table 6 below shows the results of the MRB statistics for the VaR models for each year. The EGARCH-in-M model is the highest in all models for every year, even reaching 580% MRB for crisis period 2001 and inciting crisis 2008 with 37.69% MRB. It is problematic for the EGARCH-in-M that in non-crisis 2005, it allots its highest MRB of 632%; meaning that the model may be exaggerative in giving VaR measures when no high risk of investment is felt. For the POTs, Static was the best performing with the highest of all years in MRB. For time-dependent POTs, TiVEx models are more conservative than the AGPOT with the former having higher MRB than the latter.

Model	Mean Relative Bias				Market Risk Capital			
	2001	2005	2008	2009	2001	2005	2008	2009
GARCH	-0.6464	-0.7910	0.0215	0.0317	0.0725	0.0340	0.0555	0.0711
GARCH-in-M	-0.6062	-0.7703	0.0778	0.0751	0.0811	0.0371	0.0588	0.0751
TARCH	-0.6534	-0.7964	0.0509	0.0333	0.0710	0.0334	0.0574	0.0722
TARCH-in-M	-0.6205	-0.7810	0.0795	0.0736	0.0781	0.0358	0.0590	0.0746
EGARCH	-0.6418	-0.7052	0.1426	0.3822	0.0662	0.0492	0.0565	0.0841
EGARCH-in-M	6.9456	7.5168	0.4297	0.4131	1.5603	1.4212	0.0731	0.1017
A-G-POT	-0.9160	-0.8597	-0.3276	-0.3027	0.0738	0.0607	0.0528	0.0523
Static POT	-0.5772	-0.6109	0.0673	-0.0452	0.0212	0.0226	0.0331	0.0462
TiVEx-POT	-0.7852	-0.7291	-0.2395	-0.1807	0.0457	0.0418	0.0379	0.0424
TiVEx-POT 2	-0.7857	-0.7290	-0.1135	-0.2340	0.0378	0.0423	0.0446	0.0427
TiVEx-POT 3	-0.7132	-0.7442	-0.1886	-0.2465	0.0512	0.0400	0.0408	0.0424

Table 6. Mean Relative Bias and Market Risk Capital for VaR Models for each Year

Accuracy: AQLF Results

For crisis year 2001, TiVEx-POT 3 model had accuracy comparable to the econometric models, better than the static POT model. EGARCH was still seen as a best-performing in covering losses in exchange rate risk. Simple TiVEx models 2 and 3 had smaller accuracy problems than their other time-dependent counterparts, generating only 0.008 of AQLF compared to 0.012 from TiVEx-POT and AGPOT.

Efficiency: Average MRC Results

In Table 6 of the paper, the Static POT model had the lowest MRC for almost all years except 2009, where TiVEx-POT had the lowest. Given that the Static POT had low to no exemptions in the forecast years, Static POT was the most efficient model for exchange rate risk. EGARCH-in-M had the same problem as outlined in the MRB, which is highly inflated risk capital for non crisis period 2005. For the time-dependent EVT model groups, TiVEx-POT models were better than the AGPOT model, since the latter has the highest MRC than the former, and still it is less-performing than the TiVEx models. Between the TiVEx models, the first of the three had a low risk capital for crisis situations yet is less-performing than the

three. TiVEx-POT 3, though had high risk capital for 2001, was relatively stable for inciting crisis 2008-2009 and had only 2 exemptions in 2008. This shows that the principle of parsimony may be at play even in TiVEx modeling philosophy.

V. Conclusions and Recommendations

In closing, the following generalizations were made by the research:

- ♦ Econometric models are, in general, are conservative, accurate, and efficient in predicting exchange rate losses through the VaR methodology. These models have lower exemptions and relatively lower capital to assign for risk, except EGARCH-in-M which exaggerates levels of risk and capital in non-crisis situations.
- ♦ In POT models, the Static method is seen as the best model than the time-dependent POTs and the econometric models in predicting and evaluating exchange rate risks. The Static POT has lower exemptions augmented with lower assigned capital and relatively high upward bias thus giving it conservatism, accuracy, and efficiency.
- ♦ Between time-dependent POT models, a parsimonious TiVEx model is the better model to account time dynamics since the model was able to show lower losses, lower exemptions, relatively high bias, and lower capital requirement than the AGPOT. The TiVEx is also more reliable especially in crisis situations since the TiVEx adjusts itself more flexibly using explanatory variables.
- ♦ A parsimonious or simple TiVEx model has potential to be a viable VaR methodology since it has kept its exemption status on the green side, and fear for extreme losses is little since a simple TiVEx model generated low AQLF values.

In lieu of this research, the following footsteps for further research are expressed:

- ♦ A problem surfaced in the MLE procedure for the TiVEx in the forecast year of 2001; an estimator reported negative variance and thus had no standard error reported. The researcher left alone the problem and used the estimate anyway, since this “Heywood case” for the covariance matrix of estimators cannot be resolved up to now. An expansion of the algorithm in MLE procedure to restrict variances would be of great help in the continuance of the research.

- ♦ As a conservative methodology, only stationary series were suggested as possible explanatory variables for TiVEx modeling since little or no literature exists that observes the plausibility of using non-stationary or co-integrating series. The properties and implications of using such series may be further studied in future researches which could better improve TiVEx methodology.
- ♦ This paper would like to serve as a stepping stone to explore more explanatory variables to value VaR through TiVEx. Not all variables used in this research were significant in contributing information on the dynamics of the test series, and more variables can still be used. TiVEx introduces a regression-style methodology in valuating VaR which can be refined through active research.

Final Words of the Researchers

We would like to give gratitude to our professor in financial econometrics, Dr. Claire Dennis Mapa for giving us this topic on the TiVEx model. He had expressed that it would be a challenge, it is a challenge we dared to take. He never gave the name “TiVEx;” it was Peter Cayton who facetiously coined the name.

We would also like to thank Gilleland and Katz for devising the extremes Toolkit®, which has made our research into TiVEx for financial econometrics much easier than it should be. It really helped us especially when the R program was the hardest programming language we would learn from scratch.

We would also like to thank the Institute for Development and Econometric Analysis, Inc., for sharing us their data on exchange rates which had helped us in furthering our research. Cayton is a student assistant of the Institute, and is in gratitude to the tutelage they had given him in understanding economics.

We would also to thank our parents who have been the inspiration of our research and our academic study. Their care and perseverance has been a lifesaver in tough situations.

The researchers would also like to thank each other since they wrote this manuscript without conducting overnight research together. The finishing of the paper is a slow building-up of success to the researchers in itself. Etched between its lines are the different fun and happy moments writing this paper.

Finally, we would like to thank the Providence, for wherever he may be, whatever he may be doing, he has made conditions suitable for the researchers to write the paper.

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APPENDIX I

	EGARCH	EGARCH-in-	GARCH	GARCH-in-	TARCH	TARCH-in-M	EGARCH	EGARCH-in-M	GARCH	GARCH-in-	TARCH	TARCH-in-M	EGARCH	EGARCH-in-	M 2008
	H 2001	M 2001	2001	M 2001	2001	2001	2005	2005	2005	M 2005	2005	2005	2008	2008	M 2008
Mean Equation															
c	-8.32E-06	8.8E-06	8.63E-06	2.89E-05	9.97E-06	3.55E-05	1.93E-05	-1.54E-05	1.59E-05	0.0000036*	1.92E-05	0.0000461*	6.1E-06	-0.122067*	-0.303161*
ar(1)	0.060831*	0.040446*	0.097927*	0.084715*	0.09826*	0.093327*	0.123916*	0.11216*	0.116468*	0.1048*	0.116342*	0.09191*	0.160102*	0.177749*	0.177749*
sqrt(GARCH)	-	0.019693*	-	0.047834*	-	0.057957*	-	0.002825*	-	0.050315*	-	0.064388*	-	-	0.002132
Variance Equation															
c	-0.01628*	0.006345	8.96E-10	1.16E-09	9.99E-10	0.057957	-0.09637*	-0.073772*	8.1E-10	8.55E-10	1.04E-09	1.04E-09	-0.122067*	-0.303161*	-
resid(-1) ²	-	-	0.600041*	0.700162*	0.531893*	0.773387*	-	-	0.347477*	0.462967*	0.396075*	0.542849*	-	-	-
GARCH(-1)	-	-	0.720011*	0.664712*	0.720252*	0.663246*	-	-	0.777237*	0.738003*	0.776172*	0.736152*	-	-	-
abs(stresid(-1))	0.002863	-0.509225*	-	-	-	-	0.097624*	1.828302*	-	-	-	-	0.102634*	0.224335*	0.224335*
(stresid(-1))	0.033326*	1.260089*	-	-	-	-	-0.054768*	-1.038517*	-	-	-	-	-0.061014*	-0.148828*	-0.148828*
resid(-1) ² (resid(-1)<0)	-	-	-	-	-0.084789	-0.1706	-	-	-	-	0.396075*	-0.183348*	-	-	-
log(GARCH(-1))	0.99148*	0.999821*	-	-	-	-	-	0.996641*	-	-	-	-	0.995233*	0.984447*	0.984447*
t dist DOF	2.7523661	2.000683	3.430388	3.09133	3.459453	3.135202	2.793676	2.001166	3.74441	3.425652	3.74717	3.48005	3.355472	3.355472	2.842416
Mean Equation															
c	1.08E-05	1.48E-05	1.25E-05	1.99E-05	1.05E-05	-1.89E-05	1.34E-05	2.09E-05	1.51E-05	2.61E-05	0.0000163*	2.78E-05	3.93E-06	-0.089257*	1.44E-05
ar(1)	0.135744*	0.133928*	0.136083*	0.134162*	0.154795*	0.179567*	0.139233*	0.13746*	0.139522*	0.137476*	0.071627*	0.130446*	0.159142*	0.162134*	0.162134*
ar(2)	-	-	-	-	-	-	-	-	-	-	-0.1634*	-0.19683*	-0.101886*	-0.090778*	-0.090778*
sqrt(GARCH)	-	0.014082	-	0.020554	-	0.006287	-	0.021892	-	0.028539	-	-	-	-	-
Variance Equation															
c	5.94E-10	6.48E-10	6.54E-10	7.07E-10	-0.130874*	-0.344912*	5.64E-10	6.02E-10	6.23E-10	6.61E-10	-0.006977*	-0.089257*	-0.121284*	-0.129543*	-0.129543*
resid(-1) ²	0.267168*	0.345076*	0.289272*	0.378031*	-	-	0.250006*	0.32357*	0.26844*	0.35381*	-	-	-	-	-
GARCH(-1)	0.809912*	0.770976*	0.809144*	0.769421*	-	-	0.817618*	0.778737*	0.816631*	0.777824*	-	-	-	-	-
abs(stresid(-1))	-	-	-	-	0.10312*	0.216076*	-	-	-	-	-0.009784*	0.08575*	0.09825*	0.098231*	0.098231*
(stresid(-1))	-	-	-	-	-0.058906*	-0.13297*	-	-	-	-	0.038239*	-0.0405*	-0.054531*	-0.051229*	-0.051229*
resid(-1) ² (resid(-1)<0)	-	-	-0.039736	-0.061278	-	-	-	-	-0.035321	-0.05809	-	-	-	-	-
log(GARCH(-1))	-	-	-	-	0.994538*	0.981248*	-	-	-	-	0.999403*	0.996584*	0.995034*	0.994372*	0.994372*
t dist DOF	4.38141	4.113831	4.400088	4.133789	3.536664	3.172094	4.612602	4.336837	4.628209	4.358223	2.847410	2.783485	3.351046	3.351046	3.536868

*significant at 0.1 alpha

Appendix Table 1. Estimates for the Econometric Models and the ARMA-GARCH Models Step of the AGPOT for 2001, 2005, 2008, and 2009

APPENDIX II

	Static 2001	A-G-POT 2001	TiVEx-POT 2001	TiVEx-POT 2 2001	TiVEx-POT 3 2001	Static 2005	A-G-POT 2005	TiVEx-POT 2005	TiVEx-POT 2 2005	TiVEx-POT 3 2005
Series	Original	Residual	Original	Original	Original	Original	Residual	Original	Original	Original
Threshold	0.01	2	0.01	0.01	0.01	0.01	2	0.01	0.01	0.01
Exceedances on Estimation Period	63	11	63	63	63	73	28	73	73	73
Exceedance Rate (base: 250 per year)	15.765765	2.752753	15.765765	15.765765	15.765765	9.084121	3.484321	9.084121	9.084121	9.084121
Scale	0.006958*	0.400001*	-	-	-	0.006578*	0.670716*	-	-	-
Shape	0.139614	0.642785	-	-	-	0.281236*	0.507727*	-	-	-
Scale Model (log link)										
Log Scale Intercept	-	-	0.844974*	1.622700	-0.678504	-	-	-0.739582*	0.278460	-0.414749
Trend	-	-	-0.624196*	-0.423703	-	-	-	-0.029855*	0.033861	-
Holiday	-	-	0.084711*	-	-	-	-	0.070523*	-	-
second Quarter	-	-	0.048014	-0.033901	-	-	-	0.003580	-0.178387	-
Third Quarter	-	-	-0.419130*	-0.643272	-	-	-	0.425825*	0.296092	-
Fourth Quarter	-	-	-0.691148*	-1.035105*	-	-	-	-0.748809*	-0.399822	-
Ln(GARCH(1,1)-QMLE)	-	-	0.182370*	0.410898	0.495393*	-	-	0.412971*	0.586920*	0.509428*
Shape Model (identity link)										
Shape Intercept	-	-	-0.839316*	-0.902121	0.135031	-	-	-0.155454*	-0.093520	0.133020
Trend	-	-	0.151250*	0.103305	-	-	-	0.092692*	0.025332	-
Holiday	-	-	-0.237038*	-	-	-	-	-0.169563*	-	-
second Quarter	-	-	0.343779**	-0.018551	-	-	-	-0.104404	-0.062188	-
Third Quarter	-	-	-0.339939*	0.059850	-	-	-	-0.592216*	-0.591231	-
Fourth Quarter	-	-	0.790864*	0.738120*	-	-	-	0.844682*	0.171541	-
Ln(GARCH(1,1)-QMLE)	-	-	0.039975*	-	-	-	-	0.071466*	-	-

*significant at 0.1 alpha

**reported negative variance in estimate

Appendix Table 2. Estimates of the EVT models for 2001 and 2005

	Static 2008	A-G-POT 2008	TiVEx-POT 2008	TiVEx-POT 2 2008	TiVEx-POT 3 2008	Static 2009	A-G-POT 2009	TiVEx-POT 2009	TiVEx-POT 2 2009	TiVEx-POT 3 2009
Series	Original	Residual	Original	Original	Original	Original	Residual	Original	Original	Original
Threshold	0.01	2	0.01	0.01	0.01	0.01	2	0.01	0.01	0.01
Exceedances on Estimation Period	78	48	78	78	78	83	55	83	83	83
Exceedance Rate (base: 250 per year)	7.103825	4.371585	7.103825	7.103825	7.103825	6.93748	4.597125	6.937479	6.937479	6.937479
Scale	0.00625*	0.023969*	-	-	-	0.006043*	0.630811*	-	-	-
Shape	0.286787*	-0.012644*	-	-	-	0.283760*	0.348615*	-	-	-
Scale Model (log link)										
Log Scale Intercept	-	-	-0.258175	0.268059	-0.453458	-	-	-0.319770*	-1.099470*	-0.466412
Trend	-	-	0.025911*	-0.001976	-	-	-	-0.002158*	-0.035701*	-
Holiday	-	-	0.003882*	-	-	-	-	0.230897*	-	-
second Quarter	-	-	0.478881*	0.576607	-	-	-	-0.308492	-0.151597	-
Third Quarter	-	-	-0.234834	0.025024	-	-	-	0.443711*	0.545956*	-
Fourth Quarter	-	-	-0.127372	-0.622162	-	-	-	-0.112294	-0.325229	-
Ln(GARCH(1,1)-QMLE)	-	-	0.526382*	0.545232*	0.496498*	-	-	0.494745*	0.378084*	0.493216*
Shape Model (identity link)										
Shape Intercept	-	-	0.479661*	-0.484691	0.111801	-	-	0.116198	-0.235938*	0.094599
Trend	-	-	-0.024261*	0.056794	-	-	-	0.043312*	0.028401*	-
Holiday	-	-	-0.134358*	-	-	-	-	-0.147741*	-	-
second Quarter	-	-	-0.784652*	-0.632476	-	-	-	0.433604	-0.083878	-
Third Quarter	-	-	0.105778	-0.299432	-	-	-	-0.573823*	-0.913783*	-
Fourth Quarter	-	-	0.034594	0.468232	-	-	-	0.359829*	0.207696	-
Ln(GARCH(1,1)-QMLE)	-	-	0.011628*	-	-	-	-	0.063533*	-	-

*significant at 0.1 alpha

Appendix Table 3. Estimates of the EVT models for 2008 and 2009

