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**Bootstrap Inference and Consistency in a
Spatial-Temporal Model**

by

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ABSTRACT

Nonparametric procedures based on the bootstrap are proposed for testing two assumptions in the course of estimating a spatial-temporal model: constant temporal effect across locations or spatial units and constant spatial effect over time. Bootstrap methods for time series and in regression analysis are used to approximate the sampling distribution of the estimators of the parameters of interest. Bootstrap confidence intervals on the temporal and spatial parameters are constructed for the test procedures. Consistency of the bootstrap estimators is established. An illustration in the agricultural setting is presented.

Key Words: spatial-temporal model, nonparametric bootstrap methods, bootstrap confidence interval, coverage probability, bootstrap consistency

1. INTRODUCTION

A spatial-temporal statistical model is a statistical characterization of exogenous, spatial, and temporal dependencies supported by data sampled from particular locations over a period of time (Cressie and Majure, 1997). The goal of this statistical modeling effort is spatial-temporal prediction which is achieved by systematically modeling the relationship between a response variable and potential explanatory variables, while accounting for spatial dependence and temporal dependence simultaneously (Zhu et al., 2005).

A spatial-temporal model for agricultural systems in a developing country was postulated and estimated by Landagan and Barrios (2007). They used the following additive model to properly characterize and understand the dynamics of agricultural production:

$$Y_{it} = X_{it}\beta + w_{it}\gamma + \varepsilon_{it}, \quad i = 1, \dots, N \text{ and } t = 1, \dots, T.$$

(1)

The response variable for location i at time t , Y_{it} , is expressed as a linear function of X_{it} , the set of covariates for location i at time t ; w_{it} , the set of variables in the neighborhood system of location i at time t ; and ε_{it} , the error term. Assumptions made in estimating this model include constant spatial effect over time and constant temporal effect across locations. Validity of these assumptions simplifies the estimation procedure via an iterative process that incorporates the Cochran-Orcutt procedure into the backfitting algorithm. This paper aims to develop test procedures in verifying such assumptions to aid in the estimation of the spatial-temporal model. While the study focused on model (1), the procedures can be generalized to any spatial-temporal model wherein the interest is in verifying whether there are constant or dynamic parameters over space and time, respectively.

Spatial-temporal data exhibit both temporal and spatial dependencies. Considering that the data involved are dependent rather than independent, this study takes a nonparametric approach based on resampling (Politis, 2003) in verifying the assumptions on model (1). The proposed inference procedures are based on nonparametric bootstrap methods. This allows the conduct of inferences without making strong distributional assumptions and without the need for analytic formulas for the sampling distribution of statistics involved (Mooney and Duval, 1993). This is also in the spirit of robustness of validity of the statistical analysis performed (Davison and Hinkley, 1997).

2. STATISTICAL INFERENCE WITH THE BOOTSTRAP

Given the following spatial-temporal model

$$y_{it} = f\left(\frac{x_{it}}{\theta_t}, \frac{\beta_t}{\alpha_t}\right) + \varepsilon_{it}$$

(2)

where $\varepsilon_{it} = g(\varepsilon_{i,t-1}, \rho_i) + a_{it}$, $a_{it} \sim N(0, \sigma_a^2)$. Landagan and Barrios (2007) assumed $\beta_t = \beta$ for all t and $\rho_i = \rho$ for all i to estimate the model (1) using an algorithm incorporating the Cochran-Orcutt procedure into the backfitting algorithm. Model (2) is additive and may assume a linear or nonlinear form. Without loss of generality, we assume that $f\left(\frac{x_{it}}{\theta_t}, \frac{\beta_t}{\alpha_t}\right) = \beta_1 x_{it}^{\beta_2}$ and the error term is AR(4). We then propose nonparametric inference procedures using the bootstrap to verify the assumptions on constant temporal effect across locations and constant spatial effect over time.

2.1 TESTING FOR CONSTANT TEMPORAL EFFECT ACROSS LOCATIONS

Given the time series in each location/spatial unit, we test the following hypotheses:

H_0 : All locations/spatial units have the same temporal effect.

H_1 : At least one location/spatial unit differs in temporal effect.

Based on the model-based resampling in the time domain (Davison and Hinkley, 1997) or AR-sieve bootstrap (Bühlmann, 1997), the following procedure for testing constant temporal effect across locations in model (1) is proposed.

Given the time series $y_{i1}, \dots, y_{it}, \dots, y_{iT}$ in each location i , suppose AR(p) given by

$$y_{it} = \phi_0 + \phi_1 y_{i,t-1} + \dots + \phi_p y_{i,t-p} + \varepsilon_{it}, \quad t = 1, \dots, T$$

(3)

with $\varepsilon_{it} \sim NID(0, \sigma_\varepsilon^2)$, is adequate.

1) Estimate the model (3) and predict y_{it} by

$$\hat{y}_{it} = \hat{\phi}_0 + \hat{\phi}_1 y_{i,t-1} + \dots + \hat{\phi}_p y_{i,t-p}, \quad t = 1, \dots, m$$

(4)

where $\hat{\phi}_1, \dots, \hat{\phi}_p$ are maximum-likelihood estimates of the corresponding parameters. Let ϕ_p be the temporal parameter of interest. Conditioning on $(y_{i,t-1}, \dots, y_{i,t-p})$, the empirical distribution of the centered residuals is

$$e_{it} \sim NID(0, MSE).$$

(5)

2) Generate k bootstrap samples from the distribution (5) say, $(e_{i0}^*, \dots, e_{im}^*)$ for each location i from a sample of size m .

3) Generate k time series for each location i , one for each bootstrap sample in step 2) using the estimated model (4). Each time series $(y_{i1}^*, \dots, y_{im}^*)$ will be simulated as follows:

i. Initialize $y_{i0}^*, y_{i1}^*, \dots, y_{i,t-1}^*$.

ii. Then $y_{it}^* = \hat{\phi}_0 + \hat{\phi}_1 y_{i,t-1}^* + \dots + \hat{\phi}_p y_{i,t-p}^* + e_{it}^*, \quad t = 1, \dots, m$.

4) Estimate the AR(p) model used in step 1) for each of the simulated time series in step 3).

- 5) Compute the estimated standard error for each of the estimated temporal parameter $\hat{\phi}_p$ in step 1) using the corresponding k bootstrap temporal parameter estimates $\hat{\phi}_{pj}^*$ generated in step 4) if the bootstrapped sampling distribution of the estimated parameter is normal or approximately normal:

$$\hat{\sigma}_{\hat{\phi}_p}^* = \left[\frac{1}{k-1} \sum_{j=1}^k (\hat{\phi}_{pj}^* - \bar{\hat{\phi}}_p^*)^2 \right]^{1/2}, \bar{\hat{\phi}}_p^* = k^{-1} \sum_{j=1}^k \hat{\phi}_{pj}^*.$$

(6)

- 6) Construct the 95%(99%) normal-approximation confidence intervals on each temporal parameter ϕ_p estimated in step 1) computing the limits as follows:

$$\hat{\phi}_p \pm z_{\frac{\alpha}{2}} \hat{\sigma}_{\hat{\phi}_p}^*$$

(7)

where $\hat{\phi}_p$ is the original point estimate from step 1) and $\hat{\sigma}_{\hat{\phi}_p}^*$ is its standard error estimate from step 5). Find the appropriate percentiles for the limits if the distribution of $\hat{\phi}_p^*$ cannot be assumed to be normal.

- 7) Compute the average of the temporal parameter estimates $\hat{\phi}_p$ in step 1).
- 8) Reject the null hypothesis that there is constant temporal effect across locations with 95% (99%) coverage probability if more than 5% (1%) of the constructed intervals fail to contain the average computed in step 7).

2.2 TESTING FOR CONSTANT SPATIAL EFFECT OVER TIME

Given the cross-sectional data in each time point, we test the following hypotheses:

H_0 : All time points have the same spatial effect.

H_1 : At least one time point differs in spatial effect.

Based on the bootstrap method for regression analysis, resampling the cases (Chernick, 1999), the following procedure for testing constant spatial effect over time in model (1) is proposed.

Given the cross-sectional data y_{1t}, \dots, y_{Nt} in each time point t , without loss of generality, suppose the regression model

$$y_{it} = x_{it} \beta_t + \varepsilon_{it}, i = 1, \dots, N \quad (8)$$

with $\varepsilon_{it} \sim NID(0, \sigma_\varepsilon^2)$, is adequate.

1) Estimate the model (8) and predict y_{it} by

$$\hat{y}_{it} = x_{it} \hat{\beta}_t, i = 1, \dots, N \quad (9)$$

where $\hat{\beta}_t$ is a vector of maximum-likelihood estimates of the regression

parameters. Now, without loss of generality, we identify the spatial parameter of interest, β_t , as associated with the lone predictor variable x_{it} .

2) Generate k bootstrap samples of N pairs of (x_{it}, y_{it}) from the N pairs of observations on the predictor and response variables in each time point.

3) Estimate the regression model (8) for all the bootstrap samples in each time point.

4) If the bootstrapped sampling distribution of the estimated parameter is normal or approximately normal,

- i. Compute the estimated standard error (6) for the estimated spatial parameter of interest $\hat{\beta}_t$ in the model (9) using the corresponding k bootstrap estimates $\hat{\beta}_{ij}^*$ generated in step 3) in each time point.
 - ii. Construct the 95% (99%) normal-approximation confidence intervals (7) on the spatial parameter of interest in 1) in each time point.
- 5) If the bootstrapped sampling distribution of the estimated parameter is not normal,
- i. Sort the k bootstrap estimates in either ascending or descending order.
 - ii. Find the appropriate percentiles to construct the corresponding intervals.
- 6) Compute the average of the estimated spatial parameter of interest in step 1).
- 7) Reject the null hypothesis that there is constant spatial effect over time with 95% (99%) coverage probability if more than 5% (1%) of the constructed intervals fail to contain the average computed in step 6).

3. ILLUSTRATION

Data on quarterly rice yield (mt/ha) of irrigated farms in seventy-one (71) locations (69 provinces and 2 cities) in the Philippines for the period 1990-2002 (52 observations per location) from the Bureau of Agricultural Statistics (BAS) are used to illustrate the proposed procedures. Figure 1 shows the different provinces in the country where the yield data were taken. The provinces are further grouped into regions and the regions into island groups (Luzon, Visayas, and Mindanao).

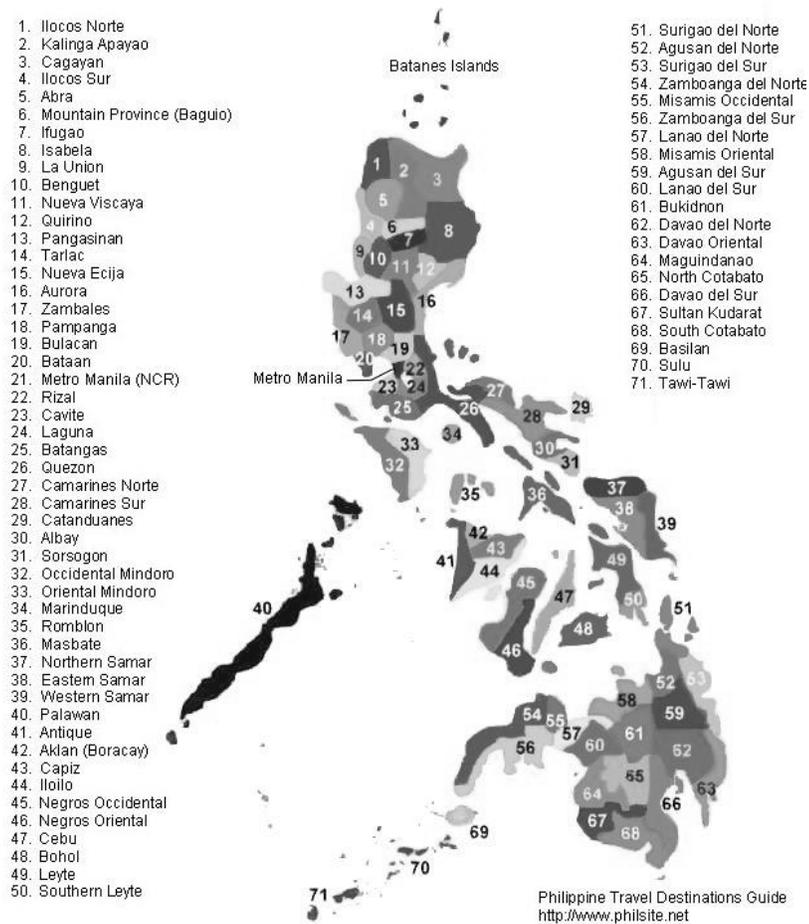


Figure 1. The Philippine map with the different provinces and the three island groups.

3.1 TESTING FOR CONSTANT TEMPORAL EFFECT ACROSS LOCATIONS

There were 71 time series on yield for the different locations (provinces), each with 52 observations. Without loss of generality, only seasonal differencing at lag 4 was used to attain stationarity of all time series prior to model estimation. This was deemed sufficient given the general behavior of these time series. Also, the autoregressive process of order 4, or AR(4), was assumed to be appropriate for this

data set based on the initial model estimations undertaken. Such process was then estimated across all locations using the model

$$y_{it} = \phi_0 + \phi_4 y_{i,t-4} + \varepsilon_{it}, \quad t = 1, \dots, 52$$

(10)

with ϕ_4 , the AR(4) coefficient, as the temporal parameter of interest.

The distribution of the original temporal parameter estimates is normal (p-value < 0.082 based on the Jarque-Bera test) with mean -0.34, median -0.36, and variance 0.0233 (Figure 2). Across locations, 77.5% of the bootstrap estimators are normal while 22.5% are approximately normal ($0.001 \leq p\text{-value} < 0.05$) based on 200 resamples per location.

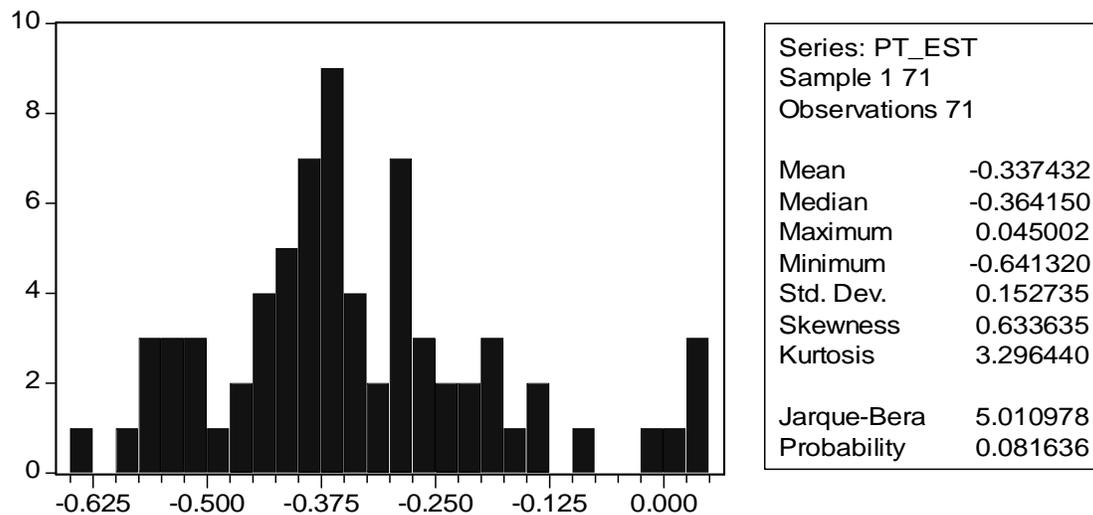


Figure 2. Histogram of the temporal parameter estimates across locations ($N = 71$).

The null hypothesis of constant temporal effect across locations, based on the mean, is rejected with 95% coverage probability (Tables 1 and 2). The provinces of Ilocos Sur (with 4th highest parameter estimate), Quirino (with 4th lowest parameter estimate), Benguet and Mt. Province (with 2nd highest and 3rd highest parameter

estimates, respectively), Romblon (with 5th highest parameter estimate), Siquijor (with highest parameter estimate), and Camiguin (with lowest parameter estimate), differ from the rest with respect to temporal effect. With 99% coverage probability, however, only Camiguin remained different from the rest. Thus, the null hypothesis is not rejected.

Table 1. Result of comparison of temporal effects across locations (N=71).

Criterion [Value]	95% C.P.	99% C.P.
Mean [-0.337432]	Reject H_0 (7)	Do not reject H_0 (1)
Median [-0.364150]	Reject H_0 (6)	Reject H_0 (2)

Note: Figures in parentheses are the number of bootstrap confidence intervals that failed to contain the mean/median.

Table 2. Bootstrap confidence intervals for the identified seven (7) different provinces in Table 1 (based on the mean with 95% coverage probability).

Province	95% C.I.	99% C.I.
Ilocos Sur	(-0.275616, 0.319342)	(-0.368958, 0.412684)
Quirino	(-0.771392, -0.348420)	(-0.837751, -0.282061)
Benguet	(-0.280637, 0.344697)	(-0.378744, 0.442804)
Mt. Province	(-0.274980, 0.326862)	(-0.369401, 0.421283)
Romblon	(-0.314831, 0.280919)	(-0.408297, 0.374385)
Siquijor	(-0.260785, 0.350789)	(-0.356734, 0.446738)
Camiguin	(-0.851428, -0.431212)	(-0.917355, -0.365285)

Based on the median, the null hypothesis is rejected with 95% and 99% coverage probabilities. Quirino was dropped from the initial list of seven based on the mean to make the new list of six based on the median. Also, the island provinces of Siquijor and Camiguin remained different from the rest even with the wider interval based on the median.

Suppose spatial dependencies (i.e., provinces/cities near each other tend to have the same behavior with respect to the effect of time) are considered. Using 95% coverage probability, only the Visayas is declared a homogeneous group (Table 3).

Table 3. Result of comparison of temporal effects by island group.

Group/Criterion [Value]	95% C.P.	99% C.P.
Luzon (N=36)		
Mean [-0.322979]	Reject H_0 (5)	Do not reject H_0 (0)
Median [-0.359779]	Reject H_0 (4)	Do not reject H_0 (0)
Visayas (N=14)		
Mean [-0.282802]	Do not reject H_0 (1)	Do not reject H_0 (0)
Median [-0.306629]	Do not reject H_0 (1)	Do not reject H_0 (0)
Mindanao (N=21)		
Mean [-0.398629]	Reject H_0 (2)	Do not reject H_0 (0)
Median [-0.388863]	Reject H_0 (2)	Do not reject H_0 (0)

Note: Figures in parentheses are the number of confidence intervals that failed to contain the mean/median.

3.2 TESTING FOR CONSTANT SPATIAL EFFECT OVER TIME

There were 52 cross-sectional data sets on yield compared, each with 71 observations. In describing the behavior of yield across locations at each time point, area harvested (ha) was the only explanatory variable available. The country-level behavior of the quarterly rice yield of irrigated farms across time is shown in Figure 3. This is reflective of the generally increasing trend in quarterly rice yield across time in a province.

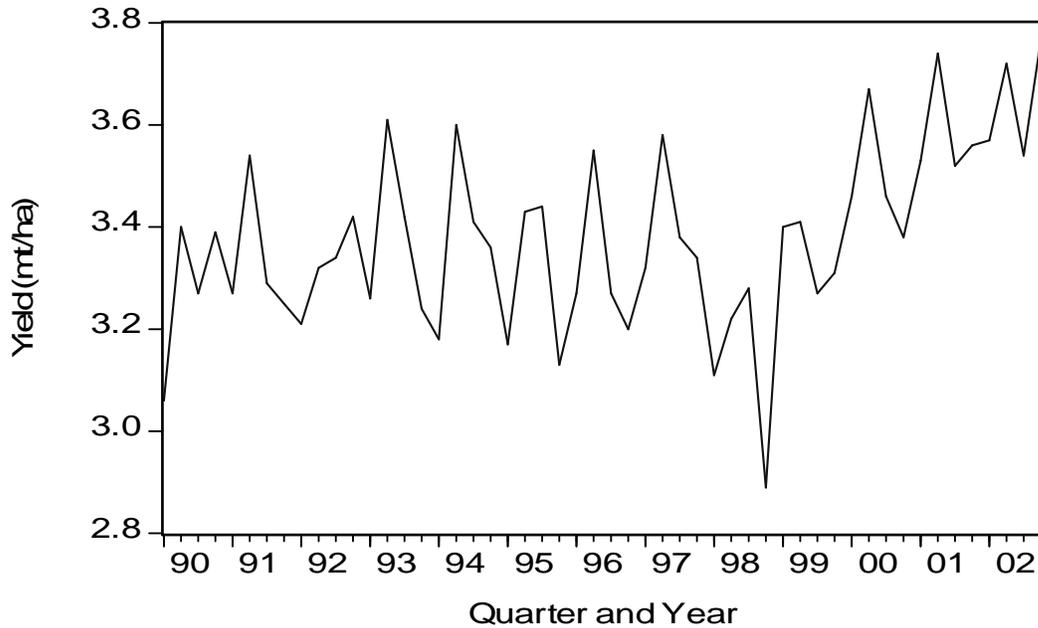


Figure 3. Quarterly rice yield (mt/ha) of irrigated farms in the Philippines, 1990-2002.

The exponential regression model was assumed to be more appropriate than the quadratic regression model for this data set based on the initial model estimations undertaken. Yield was then estimated across all time points using the model

$$y_{it} = \beta_1 x_{it}^{\beta_2} e^{u_{it}}, i = 1, \dots, 71$$

(11)

where y_{it} is the quarterly rice yield of irrigated farms in location i at time t , x_{it} is the area harvested in location i at time t , e is a constant term that is approximately 2.718, u_{it} is the disturbance term in location i at time t , and β_1, β_2 are the parameters to be estimated.

Assuming further that the exponential regression model is an accurate description of the production function, the parameter β_2 is of particular interest

(Kelejian and Oates, 1974). It indicates whether there typically exists decreasing ($\beta_2 < 1$), constant ($\beta_2 = 1$), or increasing returns to scale ($\beta_2 > 1$).

The distribution of the estimated parameter of interest, $\hat{\beta}_2$, in all time points (in step (1) of the proposed procedure) is not normal (Figure 4). At each time point, however, all estimated parameters are exactly normal at the 5% level of significance (as represented by Figure 5) except for seven (7) time points, based on 200 bootstrap samples. Those of the first quarter of 1990, 1998, and 2000 and second quarter of 2001 are approximately normal (with approximate p-values of 0.03, 0.02, 0.01, and 0.03, respectively).

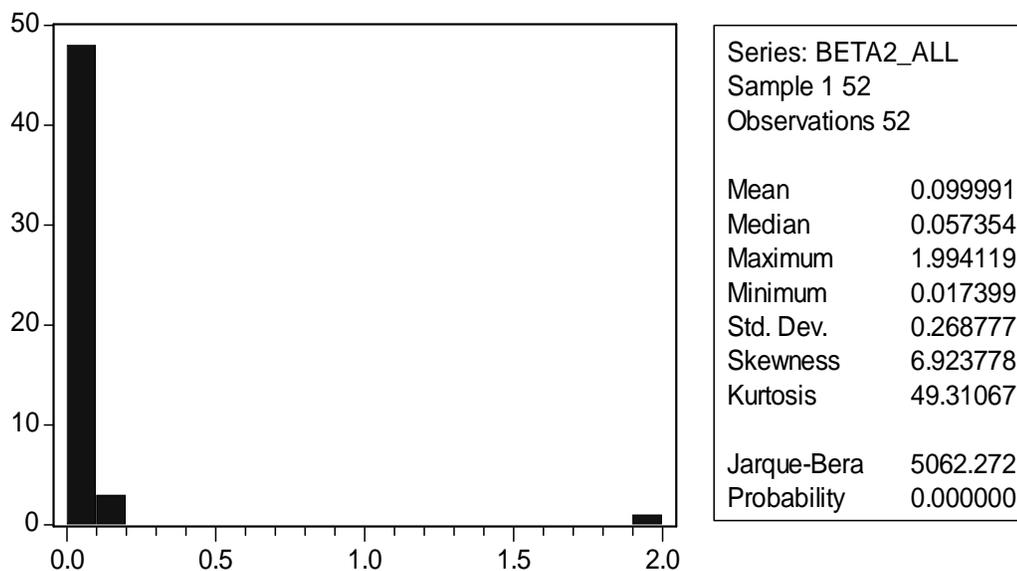


Figure 4. Histogram and related statistics of the estimated spatial parameter over time.

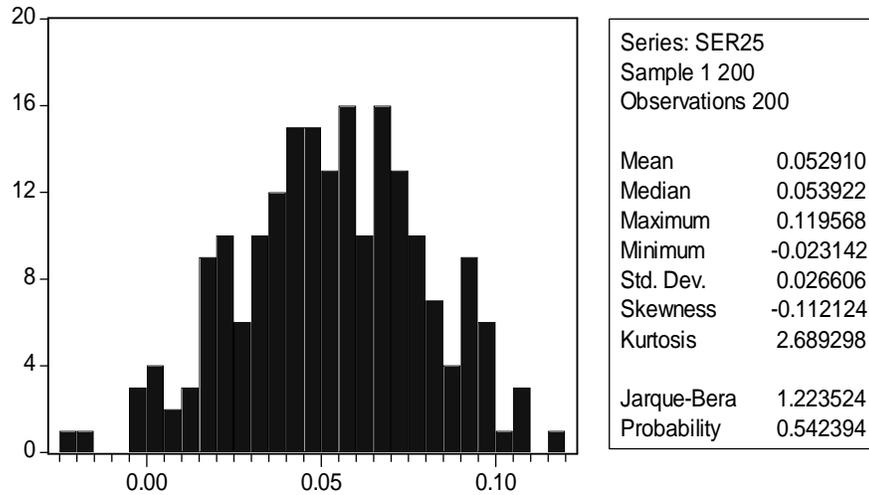


Figure 5. Typical histogram of the normal estimated spatial parameter from 200 bootstrap samples.

The non-normal estimated parameters are those of the second quarter of 1995 (negatively skewed), fourth quarter of 1992 (positively skewed), and fourth quarter of 2000 (degenerate condition; Figure 6). This last case is a case of bootstrap inconsistency or “bootstrap failure” with the parameter for this time point on the upper boundary of the parameter space defined by a linear inequality constraint (i.e., $\beta_2 > 1$) as pointed out by Andrews (2000).

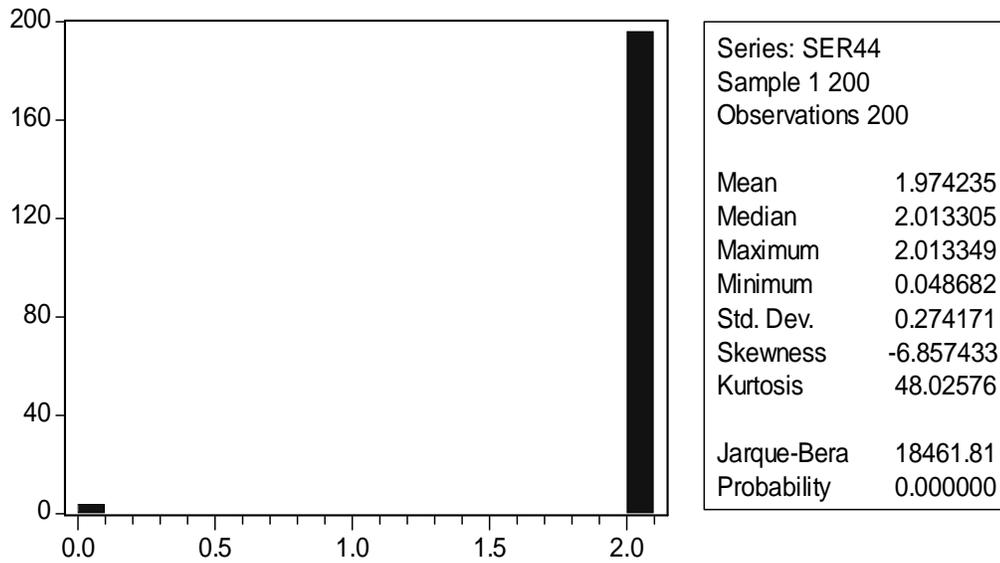


Figure 6. Histogram of the non-normal estimated spatial parameter of the fourth quarter of 2000 from 200 bootstrap samples.

The null hypothesis of constant spatial effect over time, based on the mean, is rejected with 95% coverage probability and 99% coverage probability (Table 4). Twenty-seven (27) or 52.9% and twenty (20) or 39.2% of the time points, respectively, differ from the rest with respect to spatial effect. Based on the median, the null hypothesis is only rejected with 95% coverage probability. The bootstrap confidence intervals of the eight (8) time points that differ from the rest with respect to spatial effect are summarized in Table 5.

Table 4. Result of comparison of spatial effects across all time points ($N = 51$).

Criterion[Value]	95% C.P.	99% C.P.
Mean [0.099991]	Reject H_0 (27)	Reject H_0 (20)
Median [0.057354]	Reject H_0 (8)	Do not reject H_0 (1)

Note: Figures in parentheses are the number of confidence intervals that failed to contain the mean/median.

Table 5. Bootstrap confidence intervals for the identified eight (8) different time points in Table 4 (based on the median with 95% coverage probability).

Time Point	95% C.I.	99% C.I.
1991:1	(.065313,.148155)	(.052317,.161151)
1994:2	(.057782,.114974)	(.048809,.123947)
1995:2	(.065267,.129006)	(.048073,.137372)
1996:2	(.060780,.119392)	(.051585,.128587)
1997:2	(.060127,.116433)	(.051293,.125267)
1999:3	(-.006229,.050959)	(-.015202,.059932)
2001:3	(.001984,.051208)	(-.005738,.058930)
2002:3	(-.000729,.047115)	(-.008235,.054621)

Suppose temporal dependencies (i.e., time points at the same time each year tend to have the same behavior with respect to the effect of location/space on rice yield) are considered. Using 95% coverage probability, only the third quarter time points are declared heterogeneous based on the median (Table 6).

Table 6. Result of comparison of spatial effects by quarter.

Quarter/Criterion [Value]	95% C.P.	99% C.P.
First Quarter ($N = 13$)		
Mean [0.068483]	Do not reject H_0 (0)	Do not reject H_0 (0)
Median [0.068338]	Do not reject H_0 (0)	Do not reject H_0 (0)
Second Quarter ($N = 13$)		
Mean [0.079162]	Do not reject H_0 (0)	Do not reject H_0 (0)
Median [0.081598]	Do not reject H_0 (0)	Do not reject H_0 (0)
Third Quarter ($N = 13$)		
Mean [0.051683]	Reject H_0 (3)	Do not reject H_0 (0)
Median [0.039674]	Reject H_0 (3)	Do not reject H_0 (0)
Fourth Quarter ($N = 12$)		
Mean [0.200636]	Reject H_0 (12)	Reject H_0 (12)
Median [0.052398]	Do not reject H_0 (0)	Do not reject H_0 (0)

Note: Figures in parentheses are the number of confidence intervals that failed to contain the mean/median.

4. BOOTSTRAP CONSISTENCY

To further strengthen the results using real data, the soundness of the inferences made based on the proposed procedures are justified by showing bootstrap consistency. In general, consistency of an estimator is a large sample property indicating that the estimator differs from the true parameter value by a very small amount as the sample size becomes large (Montgomery et al., 2001). Beran (1997) pointed out that the bootstrap is said “to work” when it is consistent (i.e., succeeds to converge correctly to the correct limit distribution).

Two approaches can be used to establish theoretically the consistency of the bootstrap estimators used in this study (i.e., for temporal and spatial parameters). The first shows that the corresponding basic estimator is consistent for estimating the parameter of interest (Lehmann, 1999). This is supported by the known theoretical properties of the basic estimator used.

Theorem 1. The basic estimators of the temporal and spatial parameters are consistent.

Proof:

Ordinary least squares (OLS) are used to estimate the most appropriate regression equation or ARMA model (Enders, 2004). The OLS estimators of the temporal and spatial parameters of interest were used to estimate the said parameters from the original sample data and similarly, from the bootstrap samples.

The method of least squares can be used to estimate the parameters in a linear regression model regardless of the form of the distribution of the errors ε (Montgomery et al., 2001). On the other hand, the method of maximum likelihood requires that the errors follow a normal distribution with the same second moment assumptions as required for the least-squares estimation. Now, the maximum-likelihood estimators (MLEs) of the temporal and spatial parameters are identical to the OLS estimators of these parameters (Montgomery et al., 2001). And the fact that MLEs are consistent estimators thereby makes these OLS estimators also consistent.

The second approach shows that the basic estimator is asymptotically normal (Bühlmann, 2002). In the process, Andrews's (2000) approach via the central limit theorem is adapted.

Theorem 2. The basic estimators of the temporal and spatial parameters are asymptotically normal.

Proof:

In each location/time point with k bootstrap samples drawn, let the bootstrap statistics $\{\hat{\theta}_1^*, \hat{\theta}_2^*, \dots, \hat{\theta}_k^*\}$ correspond to the k temporal/spatial parameter estimators of θ . Let $\hat{\theta}$ be the original temporal/spatial parameter estimate (also the basic estimator) of θ for this location/time point.

By bootstrapping this location/time point, the $\hat{\theta}_j^*$'s are independent and identically distributed as

$$\hat{\theta}_j^* \sim F(\theta, \sigma^2), j = 1, \dots, k, \sigma^2 < \infty$$

Let $\bar{\hat{\theta}}^* = \frac{1}{k} \sum_{j=1}^k \hat{\theta}_j^*$, then by the central limit theorem, we have

$$\sqrt{k}(\bar{\hat{\theta}}^* - \theta) \xrightarrow{D} N(0, \sigma^2)$$

This implies that the mean of the k parameter estimates, $\bar{\hat{\theta}}^*$, is asymptotically normal.

Now, the estimated bias of $\hat{\theta}_j^*$, as another point estimator of θ , is:

$$\text{Estimated Bias}(\hat{\theta}_j^*) = \hat{\theta}_j^* - \bar{\hat{\theta}}^*.$$

This makes it possible to show the unbiasedness of $\hat{\theta}_j^*$ for θ or adjust its bias for θ .

This implies that $\hat{\theta}_j^*$ is also asymptotically normal.

Thus, the bootstrap estimators of the temporal and spatial parameters can be taken to be asymptotically normal. Consequently, their basic estimators must also be asymptotically normal.

Also, the Gauss-Markov theorem holds that OLS estimators will be normally distributed if the model's error is normally distributed, and the central limit theorem assures us that this error will be normal if the sample size is "large" (Mooney and Duval, 1993). These theorems clearly make the OLS estimators (the basic estimators used) of the temporal and spatial parameters asymptotically normal.

5. CONCLUSIONS

The proposed test procedures are in support of the optimal estimation of any spatial-temporal model. The same procedures can also be used to examine closely more specific temporal and spatial dependencies that may be present in the data. No assumptions are made on the sampling distributions of the statistics used in

hypothesis testing. The bootstrap is used to approximate these sampling distributions. The implementation of the said procedures is relatively simple and straightforward despite being computer-intensive with the incorporation of bootstrap methods. For as long as the bootstrap is consistent, the proposed test procedures are expected to yield reliable results.

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