



SCHOOL OF STATISTICS
UNIVERSITY OF THE PHILIPPINES DILIMAN



WORKING PAPER SERIES

Robust Estimation of Time Series Models with Volatility

by

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UPSS Working Paper No. 2010-14
September 2010

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Abstract

Volatility in time series data is often analyzed by incorporating a conditionally heteroskedastic variance component into the model. While in-sample prediction is often satisfactory, out-sample prediction is not necessarily acceptable. Block bootstrap and AR-sieve is combined with the forward search algorithm for the robust estimation of the model for the stationary part of the time series. Using simulated data, the proposed estimation method generated robust estimates of the parameters of the time series model when there is a temporary volatility in the data.

Keywords: block bootstrap, AR-sieve, forward search algorithm, time series model, volatility

1. Introduction

Volatility has been recognized as one of the stylized facts in time series data. Certain shocks not only caused irregular fluctuations in time series, but sometimes a more complicated distortion on the structure (either mean or variance-related). In economic time series for instance, financial crisis and other exogenous events have a complicated effect on both the mean and variance of the time series, for some, it is very short-term (temporary), for others, it is lingering (or persistent).

A time series $[y_t]$, is weak stationary (or simply stationary) if the $E(y_t) = \mu$ and $V(y_t) = \sigma^2$, both are constant over time. (Enders, 1995, p.68) pointed out that stationarity allows the moments of the time series to be approximated by long time averages based on a single set of realizations alone. Volatility often affects the fluctuations in the variance, $V(y_t) = \sigma_t^2$, so that the assumption of heteroskedasticity is violated. A more general form of volatility is represented by conditionality upon an exogenous variable $[x_t]$, i.e., $V(y_t/x_t) = x_t^2 \sigma^2$. In the second type of volatility also called conditional heteroskedasticity, variance of $[y_t]$ also depends on variance of $[x_t]$, so that sudden aberration in the variance of $[x_t]$ is also reflected in the variance of $[y_t]$.

Modeling of conditional heteroskedasticity has been thoroughly explored in the literature, models like autoregressive conditional heteroskedastic (ARCH) model, and generalized autoregressive conditional heteroskedastic (GARCH) model, and further generalizations and extensions has been proposed. While these models aptly captured the behavior of a given realization of the time series, (Politis, 2007) noted that ARCH/GARCH models for financial time series have been criticized for their poor performance in volatility prediction. Volatility in some time series is temporary and short-lived. Estimation based of squared loss minimization is easily affected by the presented of volatility, and as a consequence, estimates are not robust. Thus, when used in prediction beyond the current realization, if that future scenario is not equivalent to the event when volatility occurred, i.e., when it reflects a more "tranquil" period, then predictions will be far from the actual values. This paper proposes a robust estimation procedure for a time series model in the present of volatility. A method similar to the one as proposed by (Campano and Barrios, 2010) will be considered. The paper is organized as follows: we present some modeling proposals for volatility, then we present the proposed estimation method, the simulation study, and results and discussion.

2. Volatility Modeling

The literature proposed a wide range of framework in modeling volatile behavior in time series, e.g, parametric, nonparametric, semiparametric, Bayesian, frequency domain, among others. Some of these models and general issues of estimation in volatility in time series data are presented below.

Given stationarity of the time series, (Posedel, 2005) proved ergodicity and strong stationarity for the conditional variance of the process. Further under stationarity, (Posedel, 2005) concluded that high order moments of GARCH(1,1) process exist and concluded that GARCH processes are heavy-tailed. (Hall and Yao, 2003) further noted that ARCH and GARCH models address the dependency of conditional second moments, important vehicle in simplifying the modeling process. The difficulty in estimation for ARCH/GARCH models was further linked by (Hall and Yao, 2003) to the heavy-tailed errors leading to the non-normal likelihood functions.

Autocorrelation leads to the non-diagonal variance-covariance matrix in time series data even with the constant mean, constant variance assumption. Relaxing the mean and variance assumptions lead at an even more complicated form of the variance-covariance matrix. Thus, in to mitigate this complexity in a model general longitudinal data, (Lin and Wang, 2009) factored the dependence structure in terms of the unconstrained autoregressive and scale innovation parameters in a multivariate regression to jointly model the mean and scale covariance. The simpler form of the likelihood function now facilitates estimation of parameters given the factored variance-covariance matrix.

A new type of multivariate GARCH was proposed by (Weide, 2002), that will parameterize large covariance matrices leaving a large degrees of freedom to facilitate parameter estimation. (Fan, et al, 2008) also proposed a multivariate volatility process based on a newly defined conditionally uncorrelated components (CUC) that is a parsimonious representation for matrix-valued processes. The method generally decomposed the high dimensional problem into several lower dimensional representations. Consistency was established and a test for the presence of CUC was proposed using the bootstrap method was provided. In a similar context, using as GARCH in mean models, (Lucchetti and Rossi, 2005) provide a much better testing framework the commonly used lagrange multiplier test (actual test size), specially when the process (GARCH) exhibits persistence in volatility.

In a high-frequency data, (Hafner, 1998) used ARCH class models to observe high persistence and no significant asymmetry coefficient. With the aid of a nonparametric model, significant asymmetry of the volatility function was further validated. Still, using ARCH/GARCH models, (Politis, 2007) noted their poor performance in volatility prediction. Furthermore, (Politis, 2007) proposed a model-free (alternative to parametric model) approach based on a novel normalizing and variance-stabilizing transformation.

To mitigate the predictive ability of volatility models, (Shi, et al, 2000) proposed a nonlinear autoregressive model and estimated this via kernels. The predictive ability over a parametric model was established. (Hurvich, 2005) also considered a semiparametric estimation of the memory parameter in a model that characterizes a persistent (long memory) proces.

3. Robust Estimation of Time Series

Suppose that a time series $[y_t]$ is represented by a general linear process

$$\Phi^p(B)(1 - B^d)y_t = \Theta^q(B)a_t \quad (1)$$

Following the method proposed by (Campano and Barrios, 2010), the time series $[y_t]$ is cut into blocks of constant length. The forward search algorithm [see for example, (Atkinson and Riani, 2000), (Cerioli, et al, 2007), or (Riani, 2004), for detailed discussion of the forward search algorithm] was modified by estimating the time series model in (1) for each for the blocks. When localized volatility (or conditional heteroskedasticity) is present in the time series, its effect can be contained in one or few of these blocks formed. Thus, by bootstrapping on the estimates obtained from each of the blocks, we can obtain a more robust estimate of the parameters.

The length of the block is crucial in potentially containing certain conditional heteroskedasticity into one or few blocks. Two methods of forming the blocks are proposed: mutually exclusive and overlapping blocks. Mutually exclusive block is desirable initially since each block can produce independent estimates of the parameters. However, this requires longer time series length and some persistent behavior of the time series may not necessarily be captured by a block. Block length can be adjusted but this will again require longer time series data.

Overlapping blocks can also be formed. However, overlapping blocks will produce estimates that are not necessarily independent. Thus, ordinary bootstrap method will not apply. In this case, AR-sieve bootstrap can be used. Dependent estimates are replicated to characterize the empirical behavior of these estimates, see (Bühlmann, 1997, 2002) for more details of the AR-sieve and other bootstrap methods for time series.

4. Simulation Studies

To evaluate the proposed estimation procedure, time series data under different scenarios were simulated. Parameters of the AR(1) and ARMA(1,1) models were modified to simulate a stationary and near-nonstationary process. Similarly, the parameters of MA(1) and ARMA(1,1) were modified to simulate an invertible and nearly non-invertible process. Table 1 summarized the data generating models used in the simulation.

Table 1. Models Used in the Simulation.

Process	Model	Type of Process
AR(1)	$(1 - 0.5B)(Y_t - 10) = a_t$	Stationary
	$(1 - 0.95B)(Y_t - 10) = a_t$	near nonstationary
MA(1)	$(Y_t - 10) = (1 + 0.5B)a_t$	Invertible
	$(Y_t - 10) = (1 + 0.95B)a_t$	near noninvertible
ARMA(1,1)	$(1 - 0.4B)(Y_t - 10) = (1 + 0.5B)a_t$	stationary and invertible
	$(1 - 0.95B)(Y_t - 10) = (1 + 0.95B)a_t$	near nonstationary and near noninvertible

Note: $a_t \sim N(0,1)$ for all the models.

Following (Lucchetti and Rossi, 2005), temporary structural change was simulated from a GARCH in mean model given by:

$$Y_t = 13 + \gamma h_t + u_t \quad (2)$$

where $u_t = \sqrt{h_t} v_t$, $h_t = \kappa + \delta h_{t-1} + \alpha u_{t-1}^2$, and $v_t \sim N(0,1)$. The following values for the parameters of model (2) were used: $\gamma = 0.5$, $\kappa = 0.1$, $\delta = 0.2$, $\alpha = 0.75$. A total of 1,560 time point were generated for each model. Assume monthly frequency of the time series data. The data generated from the models in Table 1 comprise the realization during a calm episode of the process. Volatility is induced by replacing a fixed number of time points at the beginning, middle and the end of the time series from the data generated from model (2). The fixed number of time points replaced were varied: 12 points (approximately 1-year persistence of volatility); 60 points (approximately 5-year persistence of volatility); and 120 points (approximately 10 years of persistence of volatility).

5. Results and Discussions

Four estimators are compared. CLS is the estimates from the conditional least squares that ignored the temporary volatility, i.e., volatility is not included into the model. The predictive ability of CLS and the proposed method are comparable. Hence, we focus the discussion on the robustness of the parameter estimates.

For the proposed method, three methods of block formation are considered. BS1: Forms overlapping segments of blocks with moving length of 1 year (12 points). BS2: Blocks are formed from overlapping segments of moving length of 5 years (60 points). BS3: Non-overlapping blocks of length 10 years each.

In a stationary AR model, robustness of the proposed model is clearly illustrated as the estimates produced are very close to the actual values of the parameters during the non-volatile periods. As the length of temporary volatility increases, the estimates becomes farther from the true values, but the overlapping moving blocks still provides estimates that are very near the true parameter values. For a near-nonstationary AR model, the length of volatile episodes still affect the parameter estimates, becomes farther from the true values as volatility becomes more persistent. The proposed estimation method is at its best comparable to CLS, in some cases, CLS is even better. For near-nonstationary AR model, the inferior performance of the estimator is the effect of cutting the time series into blocks. The near-nonstationarity of the series is exaggerated to appear to be non-stationary in a local setting captured by the blocks, similar to the observations by (Dumanjug, et al, 2010). See Tables 2 and 3 for details.

Robustness of the estimates from the proposed method in an MA model is exhibited both in Tables 4 and 5. CLS estimates becomes farther from the true values as more time points are involved in the volatile segments. Estimates from the proposed method do not deviate much from the true parameters even as more points exhibit the volatile behavior. For near non-invertible MA models, the proposed method still produced estimates that are closer to the true parameter values than what is produced by CLS.

The estimates from a stationary and invertible ARMA model are given in Table 6, while those from a near non-stationary and near non-invertible ARMA are given in Table 7. Estimates

from the proposed estimation method are closer to the actual values for both the AR and MA parameters in Tables 6 and 7.

Table 2. Parameter Estimates from Data Simulated from $(1 - 0.5B)(Y_t - 10) = a_t$.

Method	Parameters					
	$\hat{\mu}$ (s.e.)	% diff	$\hat{\phi}$ (s.e.)	% diff	MAPE	MSE
(a) 1 year temporary volatility						
CLS	10.17 (0.062)	1.70	0.58 (0.021)	16.44	8.07	1.04
BS1	10.10 (0.017)	1.04	0.50 (0.008)	0.90	8.10	1.05
BS2	10.15 (0.041)	1.45	0.52 (0.017)	3.22	8.10	1.05
BS3	10.19 (0.056)	1.87	0.54 (0.026)	8.50	8.10	1.04
(b) 5 years temporary volatility						
CLS	10.45 (0.096)	4.47	0.71 (0.018)	42.65	8.44	1.18
BS1	10.26 (0.042)	2.60	0.51 (0.013)	1.58	8.59	1.29
BS2	10.30 (0.107)	2.96	0.51 (0.03)	2.18	8.59	1.28
BS3	10.38 (0.175)	3.84	0.54 (0.043)	8.79	8.57	1.25
(c) 10 years temporary volatility						
CLS	10.90 (0.145)	9.04	0.76 (0.016)	52.65	9.19	1.85
BS1	10.68 (0.11)	6.81	0.48 (0.012)	4.96	9.90	2.23
BS2	10.78 (0.269)	7.81	0.42 (0.032)	15.49	10.35	2.37
BS3	10.90 (0.468)	8.97	0.38 (0.044)	23.15	10.81	2.49

Note: CLS – Conditional Least Squares
 BS1 – overlapping segments moving a length of 1 year
 BS2 – overlapping segments moving a length of 5 years
 BS3 – non-overlapping 10-year block

Table 3. Parameter Estimates from Data Simulated from $(1 - 0.95B)(Y_t - 10) = a_t$.

Method	Parameters					
	$\hat{\mu}$ (s.e.)	% diff	$\hat{\phi}$ (s.e.)	% diff	MAPE	MSE
(a) 1 year temporary volatility						
CLS	10.69 (0.473)	6.90	0.94 (0.009)	1.41	10.25	1.61
BS1	4.17 (3.014)	58.32	0.92 (0.006)	3.00	10.45	1.89
BS2	6.51 (3.371)	34.92	0.92 (0.013)	3.13	10.18	1.72
BS3	10.61 (0.857)	6.13	0.94 (0.014)	0.82	10.19	1.61
(b) 5 years temporary volatility						
CLS	10.52 (0.328)	5.25	0.91 (0.011)	4.48	9.63	1.53
BS1	10.07 (0.209)	0.66	0.89 (0.006)	5.94	9.63	1.54
BS2	10.32 (0.397)	3.25	0.89 (0.015)	6.47	9.70	1.54
BS3	9.97 (0.608)	0.29	0.89 (0.024)	6.73	9.66	1.54
(c) 10 years temporary volatility						
CLS	11.92 (0.435)	19.19	0.92 (0.01)	2.78	10.80	1.92
BS1	12.12 (0.337)	21.16	0.87 (0.016)	8.80	11.83	1.96
BS2	12.20 (0.88)	22.01	0.82 (0.051)	13.29	13.02	2.05
BS3	11.01 (1.266)	10.08	0.80 (0.101)	16.29	13.10	2.15

Note: CLS – Conditional Least Squares
BS1 – overlapping segments moving a length of 1 year
BS2 – overlapping segments moving a length of 5 years
BS3 – non-overlapping 10-year block

Table 4. Parameter Estimates from Data Simulated from $(Y_t - 10) = (1 + 0.5B)a_t$.

Method	Series3
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	$\hat{\mu}$ (s.e.)	% diff	$\hat{\theta}$ (s.e.)	% diff	MAPE	MSE
(a) 1 year temporary volatility						
CLS	10.09 (0.041)	0.88	-0.47 (0.022)	6.96	8.53	1.22
BS1	10.04 (0.012)	0.41	-0.48 (0.006)	3.42	8.50	1.22
BS2	10.06 (0.029)	0.63	-0.48 (0.014)	3.96	8.51	1.22
BS3	10.09 (0.039)	0.88	-0.48 (0.022)	4.04	8.53	1.22
(b) 5 years temporary volatility						
CLS	10.46 (0.053)	4.59	-0.59 (0.02)	18.66	9.57	1.74
BS1	10.27 (0.058)	2.70	-0.51 (0.006)	1.35	9.45	1.78
BS2	10.33 (0.138)	3.30	-0.51 (0.013)	1.23	9.50	1.77
BS3	10.45 (0.243)	4.54	-0.51 (0.022)	2.24	9.62	1.76
(c) 10 years temporary volatility						
CLS	10.86 (0.061)	8.59	-0.58 (0.021)	15.62	10.49	2.33
BS1	10.65 (0.097)	6.53	-0.47 (0.01)	5.83	10.42	2.41
BS2	10.74 (0.259)	7.44	-0.44 (0.035)	12.47	10.66	2.43
BS3	10.85 (0.443)	8.53	-0.37 (0.055)	25.25	11.13	2.52

Note: CLS – Conditional Least Squares

BS1 – overlapping segments moving a length of 1 year

BS2 – overlapping segments moving a length of 5 years

BS3 – non-overlapping 10-year block

Table 5. Parameter Estimates from Data Simulated from $(Y_t - 10) = (1 + 0.95B)a_t$.

Method	Series1					
	$\hat{\mu}$	%	$\hat{\theta}$	% diff	MAPE	MSE

	(s.e.)	diff	(s.e.)			
(a) 1 year temporary volatility						
CLS	10.07 (0.053)	0.73	-0.83 (0.014)	12.26	8.89	1.29
BS1	10.03 (0.022)	0.29	-0.88 (0.006)	7.78	8.80	1.30
BS2	10.06 (0.041)	0.57	-0.87 (0.012)	8.80	8.83	1.29
BS3	10.09 (0.054)	0.91	-0.87 (0.021)	8.39	8.85	1.30
(b) 5 years temporary volatility						
CLS	10.40 (0.057)	3.99	-0.72 (0.018)	24.69	9.73	1.74
BS1	10.21 (0.055)	2.12	-0.84 (0.011)	12.10	9.38	1.84
BS2	10.27 (0.132)	2.74	-0.83 (0.026)	13.07	9.42	1.82
BS3	10.41 (0.212)	4.10	-0.84 (0.039)	11.41	9.54	1.85
(c) 10 years temporary volatility						
CLS	10.93 (0.067)	9.35	-0.61 (0.02)	35.98	11.26	2.71
BS1	10.71 (0.105)	7.07	-0.78 (0.021)	18.22	10.71	3.07
BS2	10.81 (0.27)	8.13	-0.74 (0.062)	21.87	10.81	2.90
BS3	10.92 (0.458)	9.20	-0.72 (0.109)	24.60	10.96	2.82

Note: CLS – Conditional Least Squares
 BS1 – overlapping segments moving a length of 1 year
 BS2 – overlapping segments moving a length of 5 years
 BS3 – non-overlapping 10-year block

Table 6. Parameter Estimates from Data Simulated from $(1 - 0.4B)(Y_t - 10) = (1 + 0.5) a_t$.

Method	Series2							
	$\hat{\mu}$ (s.e.)	% diff	$\hat{\phi}$	% diff	$\hat{\theta}$ (s.e.)	% diff	MAPE	MSE

			(s.e.)		(s.e.)			
(a) 1 year temporary volatility								
CLS	13.05	30.51	0.94	0.84	-0.70	26.74	94.26	1.43
	(0.64)		(0.009)		(0.018)			
BS1	13.44	34.37	0.95	0.28	-0.94	0.76	98.61	1.81
	(1.568)		(0.002)		(0.011)			
BS2	10.53	5.31	0.95	0.06	-0.90	5.54	94.39	1.60
	(1.017)		(0.004)		(0.04)			
BS3	10.83	8.34	0.95	0.17	-0.84	11.53	94.76	1.50
	(1.398)		(0.01)		(0.074)			
(b) 5 years temporary volatility								
CLS	12.33	23.32	0.95	0.41	-0.12	87.60	22.47	2.62
	(0.854)		(0.008)		(0.026)			
BS1	13.71	37.08	0.95	0.00	-0.76	19.83	35.53	5.45
	(1.841)		(0.003)		(0.041)			
BS2	11.06	10.58	0.95	0.30	-0.74	22.63	33.76	5.05
	(0.881)		(0.007)		(0.096)			
BS3	11.10	11.00	0.94	1.05	-0.70	25.92	31.74	4.60
	(1.127)		(0.014)		(0.143)			
(c) 10 years temporary volatility								
CLS	10.45	4.48	0.95	0.29	-0.09	90.23	54.42	2.37
	(0.782)		(0.008)		(0.026)			
BS1	8.48	15.24	0.93	2.25	-0.64	32.38	46.09	3.71
	(0.694)		(0.012)		(0.046)			
BS2	9.60	4.05	0.87	8.00	-0.65	31.06	41.35	3.70
	(1.111)		(0.047)		(0.105)			
BS3	10.12	1.20	0.82	13.54	-0.67	29.74	52.97	3.77
	(1.597)		(0.115)		(0.175)			

Note: CLS – Conditional Least Squares
BS1 – overlapping segments moving a length of 1 year
BS2 – overlapping segments moving a length of 5 years
BS3 – non-overlapping 10-year block

6. Conclusions

In the presence of temporary volatility, a hybrid of the modified forward search algorithm and the bootstrap method can produce robust estimates of the parameters of the model during non-volatile episodes. Overlapping blocks are better than the non-overlapping blocks of time series in replicating the time series for bootstrap re-sampling. Overlapping blocks can also be advantageous in time series with relatively short length. Estimates from this method can be used in predicting out-of-sample points in the time series, conditional on the non-occurrence of volatility at those points. This will address the issue of poor prediction among volatility models noted in the literature.

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